Advances in Operations Research Models Used in the Gold Mining Industry

by

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DEDICATION

I dedicate this dissertation to the souls of my beloved late parents.

Your unconditional love has always been my inspiration.

To you, I dedicate this work.

May ALLAH bless your souls and grant you the highest state in Jannah.

I love and miss you.

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ABSTRACT

The topic addressed in this dissertation is a set of economically important operational problems in the gold mining industry that are solved using mathematical models of operations research. More specifically, the main objective of this thesis is to formulate and evaluate decision support models for three important diverse challenges which were found to exist at an underground gold mine in Northwestern Ontario: Newmont Goldcorp's Red Lake Gold Mine. The challenges discovered at Red Lake Gold Mine are not peculiar to that location but are economically relevant to the underground gold mining industry in as whole. The mine at Red Lake provided a deeper understanding of the problems and data sets.

The challenges modeled and solved in this dissertation are: i. minimizing freshwater used in the processing of gold ore; ii. optimizing ore-waste material flow in an underground gold mine; and iii. optimal dispatching of trucks and shovels in an underground gold mine. Each of the three problems was treated with a formulation of the model which is innovative and the evaluation of the results of each case study showed that improved decisions can result when these models are used.

This dissertation shows that, for a single gold mine, problems of major economic importance can be found, innovatively modeled, and solved using the methods of operations research. In addition, since these problems are not peculiar to one gold mine, but are found in other gold mines, the innovation of this dissertation is relevant to the underground gold mining industry as a whole and therefore constitutes a minor but important advance in the practical knowledge in this industry.

Keywords: underground gold mine, operations research, optimization models.

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Chapter 1: General Introduction

1.1 Introduction

At the time of this writing, the U.S. Geological Survey estimates that the average American-born citizen will need millions of pounds of fuels, minerals, and other extracted resources in his or her lifetime (Maus et al. 2022). Mining is a critical global industry, spanning all but one continent (Antarctica), with the highest-producing countries being China, the United States, Russia, Australia, and India (Dutta et al. 2016). Increasingly, this demand is driving mining companies to explore and pursue deeper mineral deposits as near-surface deposits deplete. Correspondingly, there has been a significant rise in industry interest in applying operations research techniques to improve underground mine planning (Newman et al. 2010).

The mining industry in Canada has been and remains a major component of the national economy, but this industry has historically created many challenges for the sustainability of the environment. This is confirmed by Hilson and Murck (2000), who provided some guidelines for mining companies seeking to operate more sustainably, and this is a series of actions that mine planners should take to enhance their operations' sustainability.. Their work is specifically focused on improved planning processes that contribute to sustainable development at the scale of the whole mining enterprise in Canada.

In order to remain competitive in a global marketplace, improvements in the efficiency of strategic, tactical, and operational decisions in the mining industry must continue to advance. Innovation in operations research models is one means that can assist advancements in this hierarchy of decisions in the mining industry (Newman et al. 2010). It is the overarching objective of this thesis to contribute to this advancement in innovative decision support models required in the mining industry.

To meet this general objective, we have formed a partnership with Newmont Goldcorp Inc. at the Red Lake Gold Mine, in Northwestern Ontario. At this mine, we identified three major operational problems that were in need of innovative operations research models. Each of these problems is treated as a stand-alone chapter in this thesis. These problems, and our modeling approach, will now briefly be introduced.

1.2 Problem 1: Minimizing the Use of Freshwater in the Processing of Gold Ore

At the Red Lake gold mine, we were informed that a new policy had been agreed upon whereby the amount of freshwater used in the processing of ore was to be reduced. This policy was to satisfy the environmental objectives the company had agreed to meet.

Freshwater is used intensively in the processing of gold ore. The gold mine delivers gold ore and waste rock to the above-ground processing facility, after which the ore is separated from the rock; i.e., it is purified through multiple stages of processing. Many of these stages use large volumes freshwater from the nearby lake. One strategy by which the reduction in the use of freshwater can be achieved is by replacing fresh-water with recycled water, wherever feasible, within the processing stages.

To explore the feasibility and benefits of this approach, a linear programming model of the optimal water allocation model was developed for use within the above-ground gold ore processing plant. The objective of the model is to minimize the use of fresh-water in ore processing, subject to maintaining the chemical feasibility of the recycled water used at each processing stage by satisfying constraints on pollutant concentration permissible at each stage.

The optimal water allocation problem had not been applied to the gold processing problem before. Perhaps this is because the environmental objectives had not previously existed to warrant it in the gold processing industry. The results of applying this new model to the problem instance at the Red Lake Mine showed a reduction in daily freshwater usage by 25.7% *versus* the current usage.

The research in Chapter 1 is significant for two major reasons. First, it is the first optimization model to be used in implementing the new environmental policy for gold mines in Canada to drastically reduce the quantity of freshwater used in their operations. Second, the solution of the model is practical because it shows exactly where, in the processing stages of gold ore, the use of recycled water can be most effectively used to reduce consumption of freshwater sources.

1.3 Problem 2: Truck and Shovel Dispatching Problem Used in Underground Gold Mine

Truck haulage is one of the largest operating costs in both above- and below-ground mining (Ercelebi 2009). Our partners at Red Lake Gold Mine were interested in our ability to formulate an optimization model that could help them reduce this major cost. At present, their planners are using spreadsheets to dispatch trucks and shovels. This is not surprising, although this general problem has been researched exhaustively for above-ground mines, it has received only occasional, case-specific treatments for underground mines, and no treatment in underground gold mines.

Hence, we formulated and evaluated an optimal truck dispatching model specifically tailored for the constraints and objectives of an underground gold mine.

The problem of assigning truck-trips in an underground gold mine, at the beginning of a shift, requires assigning a set of truck-trips and shovels to a set of mining levels. Each level has: i) a fixed supply of gold-ore available to be loaded in a given shift; ii) different grades of gold ore; and iii) different transportation costs arising from different slopes and distances to be covered by the trucks. The solution requires that multiple objectives be met: i) removing the greatest mass of gold (in ounces) possible in a shift; ii) minimizing total transportation costs; iii) minimizing the total number of shovels required; and iv) selecting the lowest cost set of truck types and truck-capacities required.

Given that this problem has multiple conflicting objectives, it was formulated as a goal programming model. This model allows the decision-maker to prioritize goals and to explore trade-offs in order to arrive at the optimal 'satisficing' solution. The model was applied to a data set provided by Red Lake Gold Mine and the results were compared to a single objective formulation of this model in which gold removed was to be maximized subject to the same constraints as the multiple objective models. The results show, among other things, that the multiple objective models produced a solution with a 14.8% reduction in transportation costs versus the single objective model.

To our knowledge, a goal programming model has not been formulated for the truck and shovel dispatching problem in underground mines and therefore constitutes an innovation in the field of operations research models for below-ground mines. Our research on this problem is significant for two major reasons. First, it presents and evaluates a solution method for the truck dispatching problem that is peculiar to the underground gold mine problem. The peculiarity of this problem is the variance in ore grade dispersed across the mine and the tremendous economic differences implicit in this variance. The trade-off between transportation costs, shovels used, and truck capacities selected is a difficult one. Second, the work has practical significance, for the model is quite easy to build and solutions can be generated in less than one minute, thus allowing planners to revise their dispatching plans in the middle of the shift should machines break down or other stochastic events require revision of an optimal dispatching solution immediately.

1.4 Problem 2: A Network Flow Model of Optimal Ore and Waste Movement in an Underground Gold Mine

The Red Lake Gold mine in Red Lake, Ontario is 70 years old, and its planners have inherited a myopically developed set of vertical and horizontal transportation corridors. The vertical corridors, or shafts, are each designated for either gold ore or waste. There is not capacity constraint on these shafts and the material is moved by gravity. The horizontal corridors require mining equipment's to move the ore or waste. The horizontal corridors have a limit on the capacity of material movement per day (in tonnes). These daily capacities on horizontal movement constitute bottlenecks on the flow of material through the mine. As a result, the weekly selection of blocks in which to operate (i.e., remove ore and waste) is constrained by the bottlenecks on the transportation of material that are dispersed through the mine. At present, the planners at Red Lake mine are using a heuristic method and spreadsheets to produce their weekly schedule of levels and blocks on which to operate within the constraints of their tactical plan. To solve this weekly scheduling problem, we formulated a new mixed-integer network flow model of the problem of weekly allocating mining operations in an underground gold mine such that the total gold mined (in ounces) was maximized subject to the transportation capacity constraints on ore and waste.

The results were compared to those of two greedy heuristic models that were designed to represent the decision-making heuristics that are currently used at the mine. It was found that the new model yielded solutions that improved upon the two greedy heuristics by 14.7% and 6.0%, respectively.

The research conducted is significant for two main reasons. First, it shows how a valuable problem in the underground gold mine can be solved using an operations research model with meaningful improvements over the current decision-making method. Second, the research in this problem shows how the overall productivity of the mine can be improved. Overall improvement in the mine's productivity occurs because the operational execution of the strategic and tactical plans of the mine can greatly constrain the productivity of the mine. The model minimizes a major operational bottleneck on productivity caused by the transportation capacity of the mine.

1.5 List of Publications:

1. Suliman E. Gliwan and Kevin Crowe, "*Reducing Total Fresh-water Use in the Gold Processing Industry Using Linear Programming Model*", 5th International Conference on Water, Energy, Food and Agricultural Technology Istanbul, Turkey, 20-23 March 2019, Geo-Sp Mag: Numéro 24, Vol-8 April 2019 (Publishing Guide, Geography & Sciences Publications-Magazine).

2. Suliman E. Gliwan and Kevin Crowe, "*Using Linear Programming to Minimize Fresh-water Use in the Gold Processing Industry*", European Scientific Journal November 2019 edition Vol.15, No.33 ISSN: 1857 – 7881 (Print) e - ISSN 1857- 7431.

Doi:10.19044/esj.2019.v15n33p22 URL:http://dx.doi.org/10.19044/esj.2019.v15n33p22

3. Gliwan S.E. & Crowe K. (2022) *A Goal Programming Model for Dispatching Trucks in an Underground Gold Mine*. ", European Scientific Journal, ESI Preprints.

https://doi.org/10.19044/esipreprint.8.2022.p181

4. Gliwan S.E. & Crowe K. (2022) A Network Flow Model for Operational Planning in an Underground Gold Mine. Mining Journal/Switzerland, URL:https://www.mdpi.com/2673-6489/2/4/39; DOI: 10.3390/mining2040039; Published: 10 November 2022.

Chapter 2
Fresh-water Problem Used in the Gold Processing Industry via Linear Programming

2.1 Introduction

Gold mines deliver gold ore to processing facility and waste rock to surface, after which the ore is separated from the rock through an 11-stage process (see Figure 2.1). The stages of ore processing require large quantities of fresh-water, which is drawn from nearby lakes or rivers. Given the greater importance placed on environmental sustainability, gold producers have become increasingly interested in reducing the amount of fresh-water required to process their ore. One strategy by which this can be achieved is by replacing fresh-water with recycled water, wherever feasible, within the 11 processing stages.

The objective of this research is to develop and apply an optimization model by which the gold processing industry can reduce its use of fresh-water by identifying, within the processing stages, where and up to how much recycled water can replace fresh-water. To achieve this objective, a linear programming model of this optimal water allocation problem was developed to minimize the use of fresh-water in ore processing, subject to maintaining the feasibility of the processing stages by satisfying constraints on pollutant concentrations. The model was applied to the RLGM gold processing facility, as a case study. The results show that the optimal solution generated by the model required 51 metric tons/hr of fresh-water versus the current use of 68.6 metric tons/hr - a 25.7% reduction in fresh-water use. This research is innovative insofar as an optimization model aimed at minimizing fresh-water usage, which has not been applied to the gold processing problem by prior researchers.

2.2 Literature Review

Fresh water is a basic natural resource used in many industries processes, especially in gold processing and other chemical processes. Hence, the increase in fresh-water consumption and the resulting pollutants from contaminated waste-water has become a major concern for those industries. Therefore, the government has mitigated these issues by imposing stricter environmental restrictions. We must highlight these important points in reviews by dividing them into two parts according to the evolution of tactical water allocation models that are used in gold processing: water system strategy, and the constraints imposed in the gold processing operations, as the following:

2.2.1 Optimization of a Water System Strategy:

Environmental policy-makers have repeatedly stressed the urgent need to improve planning methods on the use of fresh-water, Ridoutt et al. (2010) and Mekonnen et al. (2016). Process industries that intensively use fresh-water are therefore pursuing innovative methods to reduce their total use of this critical natural resource, Gleick and Palaniappan (2010) and Koppola et al. (2004). One promising strategy by which the use of fresh-water can be reduced in process industries is that of mathematical optimization models Izquierdo et al. (2004), Molle et al. (2008), and Khan et al. (2018). In the same way, Bagajewicza et al. (2000), Bagajewicza & Savelski et al. (2001), and Saeedi et al. (2006) confirmed that access to an investigation of water system strategy will be beneficial to mitigate the environmental constraints imposed on chemical processing operations, and others, to reduce fresh-water usage as well as to contribute to increasing used-water consumption in their processes. Therefore, some studies and research have addressed the water system strategy in gold processing or chemical units. Wang, et al. (1994b) confirmed that the treatment of fresh-water after use in the gold processing, as well as the contamination resulting in their processes, are important factors to reach an optimal water system strategy and reduce environmental restrictions. Koppola et al. (2004) discovered many benefits from implementing a water system strategy into processing operations, especially those that involve chemical processes. They indicated that a water system strategy reduces environmental constraints by using recycled water instead of fresh-water. Zaman et al. (2015) presented an important review about the benefits and challenges of water management system strategies and asserted that the concept has been implemented generally in different phases of processing operations and used-water management systems. Hence, a water system strategy and contaminated water policy outlining ways to take advantage of recycling used-water in the processes would be a beneficial strategic organizational goal and would assist decision-makers to improve their company's strategy to implement a water policy for output of their processes. In addition, Pietzsch et al. (2017) presented valuable literature reviews on the subject of "zero waste water" in their study of 102 published articles on this topic. They all recommended that there are financial, economic, and environmental benefits, as well as advantages to industrial processes by implementing a water management policy and making it a strategic goal.

2.2.2 Optimization Programs Model:

In this part, we focused on the reviews related to optimization models used to solve the water allocation system problem in the processing operations of chemical industries, which has been intensively researched. Since the 1970s, many industries suffered from a water management problem that involved increased water consumption, as well as seeking an efficient way to take advantage of recycling used-water in the processes. Several researchers, e.g., Schultz et al. (1974), Hospondarec et al. (1974), Anderson (1977), and Sane et al. (1977) have developed optimization formulation models to solve the fresh-water problem used within the processing stages and manufacturing industries. They identified where, and up to how much, recycled water can replace fresh-water consumption during processing stages. They explained the importance of using mathematical programming models to solve water allocation management problems and to increase the contribution of recycled used-water throughout manufacturing processes as an alternative during cleanup and processing processes. They also discussed the influence of the concentration of pollutants on inlet and outlet in each process to minimize water consumption in petroleum industries and chemical processes. Research carried out by Bagajewicz et al. (2000, 2001 and 2002), Karuppiah et al. (2006), Saeedi and Hosseinzadeh (2006), Klemes et al. (2010) on particular water-intensive industries has also been carried out, e.g., in petroleum refining. Takama et al. (1980) and Alva-Argaez (2007) studied aluminum processing. Deng et al. (2009) conducted research on pulp and paper production, as did Lovelady et al. (2007). To the best of our knowledge, the gold processing industry has not been evaluated by the application of a mathematical optimization model formulated to minimize its use of fresh-water. This gap is significant, for the gold processing industry is water-intensive as confirmed by Mudd, et al. (2007). Joe & Pickett (1974) and Turcotte et al. (1986) reiterated that the recycling of used-water in Canadian gold mines helped tremendously in purifying the contaminants such as Cyanide, Uranium, Zinc, Sulfur, Carbon, Nickel, and Magnesium, which cause extreme damage to the environment. Jezowski et al. (2010) presented a comprehensive review on the development of fundamental optimization models, abstracted from the particular constraints of a given type of industrial process.

Based on the above, it can be said that a water system strategy has become a prime target for gold mining industries. Hence, reaching an optimal fresh-water consumption used in gold mining processes and replacing it with the used-water is a strategic objective, and is being treated as a genuine goal for all gold mining companies.

2.3 Methods

In the Methods section, we first present the problem definition and the case study parameters; secondly, we describe the mathematical formulation of the model used to solve the problem.

2.3.1 Formulation of The Model

The model used in this problem is categorized as an optimal water allocation model. Several such models have been formulated for this problem by researchers working on fundamental issues in the process industries (e.g., Bagajewicz et al. 2000, and Koppol et al. 2004); the model presented below is derived from this previous work. Therefore, the innovation in this work is not in the formulation of the model, but in its application to the gold ore processing problem, and the results thereof. The mathematical formulation is presented below:

Indices and Sets

j, J = *index and set of processes.*

h, H = *index and set of processes in which pollutant are added.*

k, K = *index and set of mass load pollutant processes.*

P_j = *set of antecedent processes whose output flow is a direct input to process j.*

R_j = *set of receiver processes whose input flow is a direct output from process j.*

Parameters

$L_{hj,}$ = *contaminant mass load of pollutant h into the process j (kg per hour).*

$C_{j,in}^{max}$ = *maximum allowable concentration of pollutants input to process j (ppm).*

$C_{j,out}^{max}$ = *maximum allowable concentration of pollutants output from process j (ppm).*

$\hat{H}$ = *% contaminant head at the processes using wastewater (kg per hour).*

Decision Variables

$F_j{}^w$ = flow of the fresh-water input to process j (metric tons/hr).

$F_{i,j}$ = flow of contaminated water from process i to process j (metric tons/hr).

Objective function

$$Minimize\ (\mathrm{Z}) = \sum_{j \in J} F_j^w \qquad [1]$$

Subject to:

$$F_j^w + \sum_{i \in \boldsymbol{R_J}} F_j = \sum_{i \in \boldsymbol{R_i}} F_{i,j} \qquad \forall_j \in J \qquad [2]$$

$$F_h^w - \frac{L_h}{C_{h\ out}^{max}} = 0 \qquad \forall_h \in H \qquad [3]$$

$$\sum_i F_{i,j}\ \left(C_{i,out}^{max} - C_{j,in}^{max}\right) - F_j^w\ .\ C_{j,in}^{max} \leq 0 \qquad \forall_j \in \bar{H} \quad i \in P_i \qquad [4]$$

$$\sum_i F_{i,j}\ \left(C_{i,out}^{max} - C_{j,in}^{max}\right) - F_j^w\ .\ C_{j,in}^{max} + L_h \leq 0 \qquad \forall_j \in \bar{H} \quad i \in P_i \qquad [5]$$

$$C_j \leq C_j^{max} \qquad \forall_j \in \bar{H} \qquad [6]$$

$$C_i \leq C_i^{max} \qquad \forall_i \in P_i \qquad [7]$$

$$F_j{}^w \geq 0 \qquad \forall_j \in J \qquad [8]$$

$$F_{i,j} \geq 0 \qquad \forall_i \in I \quad \forall_j \in J \qquad [9]$$

The objective function of this model, as shown in Equation [1], is to minimize the total fresh-water input into all gold processing stages. Hence, each process uses water flow (fresh-water or used-water, or both) in process i to process j, which includes contaminant mass load Lh and the maximum concentration rate of inlet (C_{in}) process and outlet (C_{out}) process (as the constraints of concentration rates). Equation [2] is a flow balance equation it ensures that, for each process, the total water flow in (metric tons/hr) equals the flow out. Therefore, this constraint described for a water process flow has two requirements: a water flow rate in the process i to process j, and a concentration rate. Equation [3] ensures that the mass load pollutants, L_h (kg.h-^{1}) contained in the flow of water out of each process does not exceed its limit C_{out}^{max}. Therefore, before calculating this Equation [3], we need to be aware of the water weight and pollutants outflow in each process L_h (kg.h-^{1}) because the concentration of mass load pollutants impacts the amounts of freshwater consumption (mg/L = ppm = g/ton). Here, we note that, in this model, the mass load is calculated using the following equation:

$$\text{Mass load of polluants } (L_h) = \frac{Waterweight*(1000L*1\ kg)*pollutantoutletmg*(1g)}{\text{hr.}*(\text{1000g}) \quad * \quad \text{L}*(\text{1000mg})} \qquad [8]$$

Hence, it is determined that for mass load L_h (kg.h^{-1}), which represents the number of pollutants concentration used for k in process j, it is volume that affects the inlet and outlet to process h, where each process ∀h belongs to $\overline{H}$ (all sat processes that use reused water and sends a contaminant mass load to the next process). Equation [4] ensures that the mass load L_h (kg.h^{-1}) of pollutants contained in the flow of water out of each process does not exceed its limit, C_{out}^{max}. This constraint is used in processes that are fed only by fresh-water H (tons/hr). When implementing this constraint using the optimization programming model, it calculates the difference between the maximum concentration outlet (C_{out}^{max}) in water process i, and the maximum concentration inlet in water process used (C_{in}^{max}). $\sum_i F_{i,j}\ \left(C_{i,out}^{max}\ -\ C_{j,in}^{max}\right) - F_j^w\ .\ C_{j,in}^{max}\ \leq 0$, for each process i to j. Where each process h belongs to H (a set of contaminants head (k.h-1) at the processes that used fresh-water (tons/hr); and where the process i belongs to P_j (all set processes which post fresh-water and waste-water to process j). Therefore, the optimal design problem can be addressed in this constraint for processes that used fresh-water. Equation

[5] ensures that the mass load L_h ($kg.h^{-1}$) of pollutants contained in the flow of water out of each process does not exceed its limit, C_{out}^{max}. This equation differs from Equation [4] in that it is used only for processes that are fed by recycled water or a mixture of fresh-water and used water. Therefore, when implementing this constraint, it calculates the difference between the maximum concentration outlet (C_{out}^{max}) in water process i, and the maximum concentration inlet in water process i (C_{in}^{max}), plus contaminant mass load L_j, because it contains pollutants concentration flow. Where each process h belongs to $\overline{H}$ (a set of contaminants head at the processes used wastewater ($k.h^{-1}$); and where process i belongs to P_j (all sat processes that post fresh-water and waste water to process j). It is noticed that the fifth constraint in Equation 5 is important because it is related to the contaminant of mass load pollutants (L_h) in each to $\overline{H}$ process set.

2.3.2 Case Study Problem

Figure 2.1 illustrates the 11-stage process by which gold is extracted from the ore that has been mined. The parameters are from our case study.

Reports published in 2013, 2015 and 2017 refer to Newmont Goldcorp Inc., in addition to recent data obtained through site visits and interviews with RLGM employees in the water system department regarding the data and information about the 11 stages in the case study. The average output of ore material gold production capacity during their gold processing stages is 1800 tons per day from ore material in order to produce between 800 to 1200 ounces of gold (1 ounce = 31.1 grams). In the case study, the current water allocation system did not allow them to control water allocation management in all their processes. Where the calculated water quantity in most processes is based on a density rate in each process, this ratio is often dependent on estimates of water used. Hence, the estimated total amount of water used in their processes (fresh-water + used-water) is estimated at 102.6 metric tons/hr. The total usage to produce the gold was 68.6 metric tons/hr of fresh-water (F^w- 67%), and 34 metric tons/hr of used-water (U^w - 33%), as shown in Figure 1.1.

This chapter's goal is to create and put into practice a strategy for a gold processor to use less freshwater by figuring out where and how much-recycled water can substitute freshwater during the various stages of processing gold ore. In order to limit the amount of fresh- water needed in the processing of gold ore, while maintaining the feasibility and viability of the various phases of the gold processing. To achieve this objective, we will utilize a linear programming model to arrive at the best solution for the water allocation problem in the case study at (RLGM).

F^w = Fresh-water **U^w** = Used-water F^{flow} = Water used in previous process (slurry + water)
P_1 = Ore crushing **P_2** = Ore grinding **P_3** = Floatation concentrate sulfide from the ore (air+ reagents)
P_4 = Thickener autoclave (Oxygen + sulfides + water) **P_5** = Autoclave (Oxygen + sulfides + water)
P_6 = Carbon in leach (CIL) to dissolve gold and absorbed by the carbon
P_7 = Leaching (dissolved gold) **P_8** = Carbon in pulp (CIP) to absorb gold from solution
P_9 = Detox (Cyanide destruction) . **P_{10}** = Paste backfill
P_{11} = Waste treatment **T_1** = Used-water tank

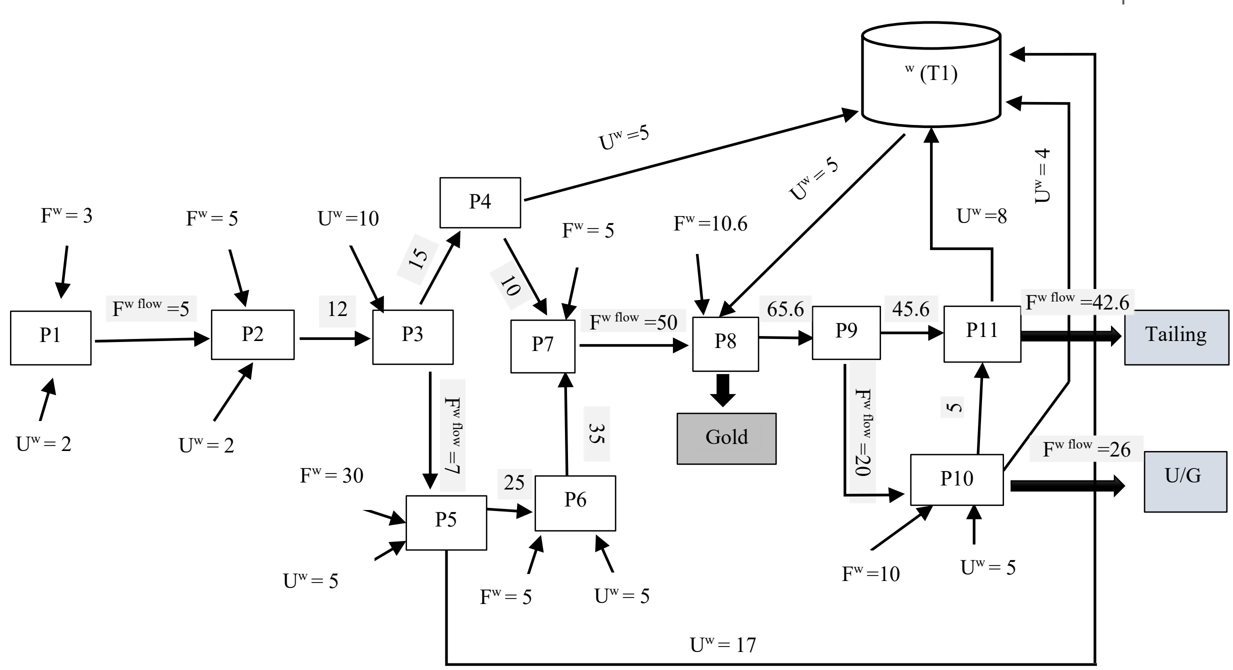

Figure 2.1: The 11-stage process by which gold is extracted from ore material.

In Figure 2.1, one can observe that two sources of water can be used at each processing stage: either (i) fresh-water (F^w), or (ii) partially recycled water (U^w). In Figure 2.1, the recycled water comes from a tank (T). Figure 2.1 also shows the parameters of fresh-water and recycled water currently used at this facility (the values in Figure 2.1 represent the flow of water in metric tons per hour). These water flow parameters are not mathematically optimal because they were decided upon at a time before when the emergence of fresh-water conservation was becoming an environmental objective. Nonetheless, the parameters outlined in Figure 2.1 are feasible, i.e., they restrict the total concentration (in parts per million) of pollutants that are allowed to enter a given processing stage (C_{in}^{max}) or exit a given processing stage (C_{out}^{max}) such that the processing that occurs at each stage is feasible. The parameters used at this facility are presented in Table 2.1.

Table 2.1: Water use and pollutant parameters currently existing in the 11 stages of ore processing at the facility used in this case study.

No. Proc.	Processes	Capacity: Water m. tons/hr					C_{in}^{max} (ppm)	C_{put}^{max} (ppm)
		F^w	U^w	F^{Flow}	Solid m.tons/hr	Total m.tons/hr		
P1	Crushing	3	2	0	80	85	30	65
P2	Grinding	5	2	7	80	92	546	1068
P3	Floatation	0	10	12	80	102	253	751
P4	Thickener	0	0	15	76	91	460	1127
P5	Autoclave	30	5	7	4	46	6256	27211
P6	Carbon in Leach (CIL)	5	5	15	4	29	720	2743
P7	Leaching	5	0	30	80	115	875	1522
P8	Carbon in Pulp (CIP)	10.6	5	35	80	130	843	1540
P9	Detox	0	0	50	80	130	663	1326
P10	Paste Backfill	10	5	10	70	95	487	1393
P11	Waste Treatment	0	0	45	10	55	382	1278
T	Used-water Tank						507	1081
	Total	68.6	34					

Table 2.1 shows that the facility currently uses 68.6 metric tons of fresh-water per hour and 34 metrics tons per hour of recycled water. The pollutant constraints at each stage of processing are also listed, as shown in Figure 2.2.

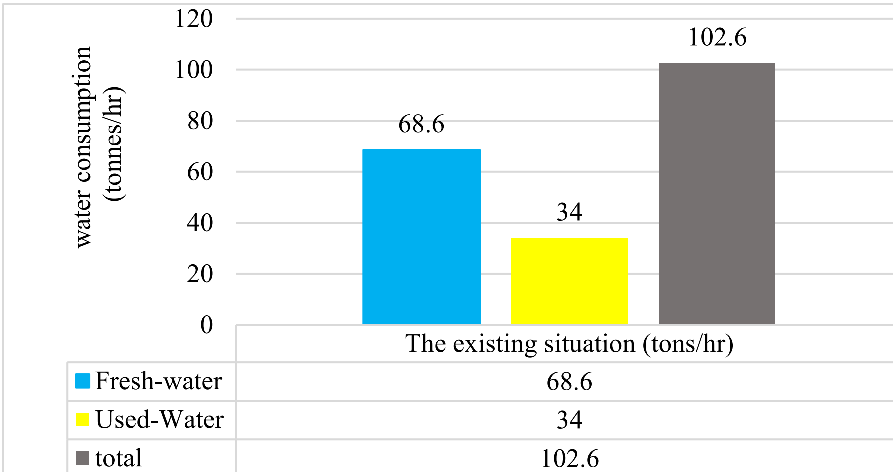

Figure 2.2: Fresh-water consumption rate and used-water in the existing situation processes, which uses one tank of used. Goldcorp Inc.'s reports (2015, 2017 & 2019).

The problem that must be solved is that of minimizing total fresh-water used per hour by replacing fresh-water with recycled water at each stage (wherever feasible) such that restrictions on C_{in}^{max} and C_{out}^{max} are satisfied. The feasible mass load of pollutants, at each process, are presented in Table 2.2, for example, to calculate (L_h) for the process (P6). Thus, all processes are calculated by using the Equation [3]. Note: (mg/L = ppm = g/ton).

$$\text{Mass load polluants } (L_h) = \frac{Waterweight*(1000L*1\,kg)\ *pollutantoutletmg*(1g)}{\text{hr.}*(\text{1000g}) \qquad \text{L}*(\text{1000mg})}$$

Given an example for process P6 (Carbon in leach CIL), refer to Table 2.2.

$$L_h\ (\text{P6}) = \frac{(25*\ (1000L*1Kg/hr)*(\ 2743\ \frac{mg}{L}*1g)}{(\text{1000g})\ (\text{1000mg})} = 68.6\ \text{kg.hr}^{-1}$$

Table 2.2: Mass load of pollutants at each stage of the ore processing problem.

No. of Process	Process	Contaminant Mass load (L_h) (kg/h)	Max. inlet of pollutant concentration C_{in}^{max} (ppm)	Max. outlet of pollutant concentration C_{out}^{max} (ppm)
P1	Crushing	0.33	30	65
P2	Grinding	12.8	546	1068
P3	Floatation	16.5	253	751
P4	Thickener	16.9	460	1127
P5	Autoclave	1006.8	6256	27211
P6	Carbon in Leach (CIL)	68.6	720	2743
P7	Leaching	53.3	875	1522
P8	Carbon in Pulp (CIP)	77.9	843	1540
P9	Detox	66.3	663	1326
P10	Paste Backfill	34.8	487	1393
P11	Waste Treatment	57.5	382	1278
T1	Used-water Tank		507	1081
	Total	1410.83	12022	41105

The solution to be investigated in this water allocation problem should be regarded as an exploration of the first step toward the reduction of fresh-water usage in this facility. This first step could be implemented at minimal cost (; although the capacity for water recycling water may need to be expanded, no additional water recycling technology would be required). Hence, this problem is one replacing a currently feasible solution with an optimal solution that would require minimal infrastructure investment.

2.4 Results

The results of applying the optimal water allocation model to the case study are presented in Figure 1.3. Hence, the optimal allocation of the quantity water consumption flow in their 11-

stage processes, as shown in Figures 2.3 and 2.4, is summarized and compared with the current allocation of fresh-water in Table 2.3.

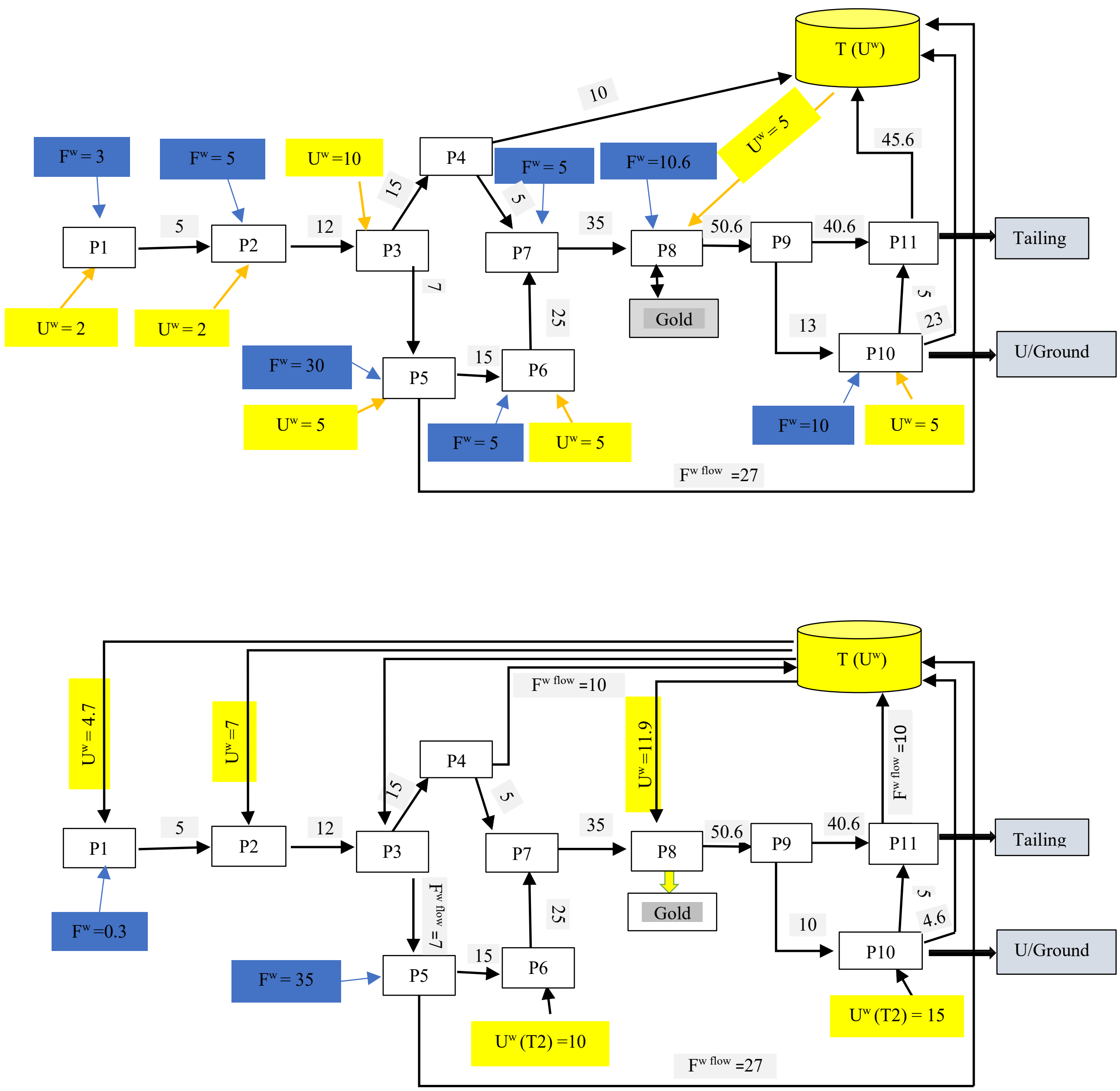

Figure 2.3: Comparison between the actual water consumption flow and the optimal solution in case study (RLGM) processes.

The above results show that reductions in fresh-water input occurred at processes 1, 2, 3, 6, 7, 8, and 10 while, on the other hand, increased fresh-water input occurred at process 5. The results also reveal that the total fresh-water used decreased from 68.6 metric tons per hour to 51 metric tons per hour – a reduction of 25.7%. In addition, the use of recycled water increased from 34 to 51.6 metric tons per hour – an increase of 51.8%. Hence, the use of the optimal water allocation model facilitated a major reduction in the current use of fresh-water in this case study.

Table 2.3: Comparison of the current versus optimal allocation of fresh-water in the 11-stage ore gold processing problem.

No. Proc	Processes	Capacity Actual water flow metric tons/hr			Actual water flow %		Optimal water flow metric tons/hr			Optimal water flow %	
		F^w	U^w	F^{Flow}	F^w	U^w	F^w	U^w	F^{Flow}	F^w	U^w
P1	Crushing	3	2	0	2.9	1.9	0.3	4.7	0	0.3	4.7
P2	Grinding	5	2	7	4.9	1.9	0	7	7	0	6.7
P3	Floatation	0	10	12	0	9.8	10	0	12	9.7	0
P4	Thickener	0	0	15	0	0	0	0	15	0	0
P5	Autoclave	30	5	7	29.2	4.9	35	0	7	34.1	0
P6	Carbon in Leach (CIL)	5	5	15	4.9	4.9	0	10	15	0	9.8
P7	Leaching	5	0	30	4.9	0	2	3	23	1.9	2.9
P8	Carbon in Pulp (CIP)	10.6	5	35	10.3	4.9	3.7	11.9	35	3.6	11.7
P9	Detox	0	0	50	0	0	0	0	50.6	0	14.6
P10	Paste Backfill	10	5	10	9.7	4.9	0	15	10	0	0
P11	Waste Treatment	0	0	45	0	0	0	0	40.6	0	0
T	Used-water Tank				0	0	0	0	0	0	0
	Total	68.6	34		67%	33%	51	51.6		49.6%	50.4%

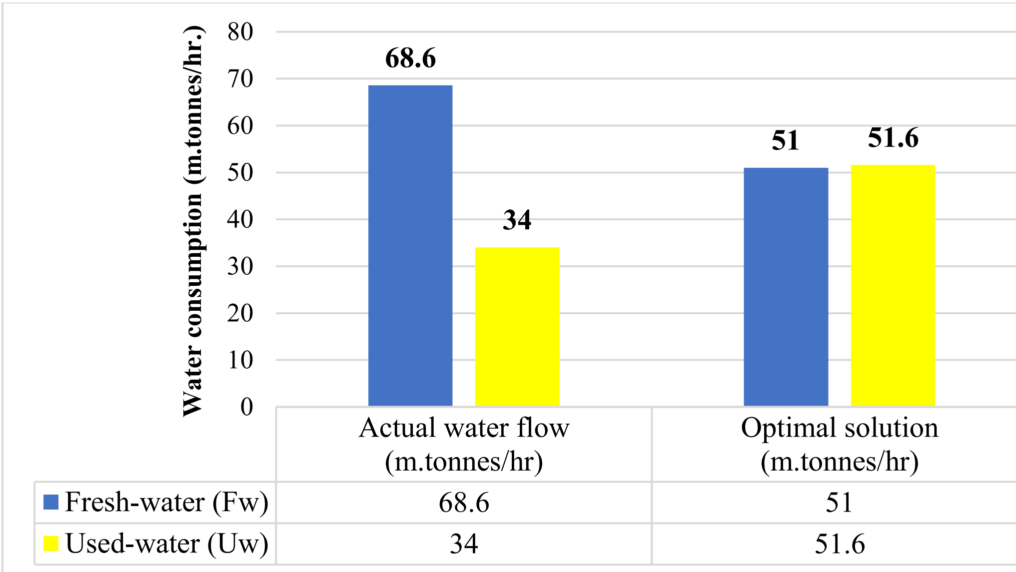

Figure 2.4: Results comparison.

The model was built and solved with Microsoft Excel's Solver using a Windows 10 operating system using an Intel CORE i7 CPU with 16 gigabytes of RAM. The solution times for all problem instances were under 20 seconds.

2.5 Discussion

Our first point of discussion addresses the question of why such a major reduction in the total use of fresh-water in this case study (25.7%) was possible. The answer lies in the historical and geographical context in which the processing facility was designed. RLGM's processing facility is over 70 years old and is located in an area surrounded by many lakes. Hence, there is no scarcity of fresh-water, nor in the era in which the design occurred -- one in which the ecological impact of excessive use of fresh-water was not fully appreciated. The lesson that this study presents, therefore, is that the global goal of an environmentally sustainable future may be advanced by optimizing the water allocation problem in older gold processing facilities.

The second point of discussion concerns evaluating the feasibility of the solution. The results show that the decrease in total fresh-water usage came at the cost of increasing recycled water usage by 51.5%. Hence, the current capacity of the processing facility's ability to recycle water must be expanded to implement this solution. Goldcorp has recently placed the efficient use of water as a top strategic goal for the corporation. Hence, the practical feasibility of this solution appears promising.

A final point of discussion concerns further useful applications of the optimal water allocation model to the ore processing problem. As noted earlier, this problem presents only the first step that can be taken to reduce the total fresh-water used in the 11-stage ore gold processing. Further steps would involve evaluating investments in water recycling infrastructure that is designed specifically for earlier and less polluted stages of the ore processing problem. At present, the recycling technology used at this facility treats all recycled water, from the most to the least contaminated, using the same technology. Therefore, the model used in this problem can be used to evaluate the benefits of using less intensive recycling technologies at earlier stages in the ore processing stages.

2.6 Conclusion

In this chapter, we presented the application of an optimal water allocation model to the problem of minimizing fresh-water use in the 11-stage gold-ore processing problem as the case study. The innovation of this work was not in the formulation of the model, but in its application to the gold ore processing problem. Although the results showed that a major reduction (25%) in the use of fresh-water is feasible, it comes at the cost of expanding the current recycling capacity of RLGM's facility by 51.5%. Hence, this study indicates that steps can be taken towards a sustainable future, which requires the efficient use of fresh-water, by allocating fresh-water more efficiently in the gold processing industry.

Chapter 3

A Goal Programming Model for Dispatching Trucks in an Underground Gold Mine

3.1 Introduction

The movement of mined ore in an underground mine typically represents between 50 to 60 percent of a mine's total operating costs (Newman et al., 2010; Fadin et al., 2017). Therefore, the dispatching of trucks in an underground mine is a daily decision of major economic consequence and warrants the use of a decision support model. In dispatching problem addressed in this paper, a set of trucks must be assigned to a set of trips and to a set of mining levels with each containing different grades of ore. The assignment of trucks must be made in such a way that four objectives are met: transportation costs are minimized, the ounces of gold retrieved are maximized, the number of shovels used is minimized, and the total number of trucks required is minimized for a given shift. Given that this problem has multiple conflicting objectives, a goal programming model is developed and tested in this paper. The objective of this paper is therefore to formulate and evaluate a goal programming model of the truck-dispatching problem for underground gold mines.

The chapter is structured as follows: first, a review of the literature relating to this dispatching problem is given. Second, the problem modeled is defined. Third, the mathematical formulation of the model is presented. Fourth, a description of the case study on which the model is to be evaluated, the Red Lake gold mine in Ontario, Canada, is presented. Finally, the results of the model are presented, and the merits of the model are observed and evaluated in the discussion.

3.2 Literature Review

The truck dispatching problem in the mining industry has received minor but consistent attention by researchers who specializes in optimization models. The problem has received wider attention for above-ground mines than for below-ground mines. Indeed, there are few studies that have been made for truck-dispatching in underground ground mines (Mahdi et al., 2014). Newman et al. (2010), in their review paper on operations research models used in mining, observed that rather than one universal dispatching model for the mining industry, there exists a great diversity of models for this problem. This is because the different types of mine structures require different objective functions and different constraints. Hence, there is no universal model of the truck dispatching problem (in either above- or below-ground mining) given the great diversity of mines structures (Newman et al., 2010). In this review, we will examine the diversity of optimization models that have been recently formulated for the truck dispatching problem in both above ground and underground mines.

Ercelebi et al. (2009) used a linear programming model to improve the truck-to-shovel dispatching system and established a method of accurately determining the optimal number of trucks. They also applied the single objective model to an open-pit coal mine in Turkey. Nehring et al. (2010) formulated a mixed integer programming model and applied it to a transportation system that used trucks and shovels in an underground mine in order to maximize net revenue within a shift. Song et al. (2013) formulated a linear programming model to solve the truck and shovel dispatch problem in an open-pit mine. This paper focuses on maximizing the total transportation (in tonnes) of ore and waste material in a given shift. Zhang et al. (2015) presented a new model of the truck dispatching problem in an open-pit iron mine by using integer programming to represent the optimal number of discrete trips for trucks to make between loading sites and dumping sites in one shift. However, their results showed reduced transportation operating costs of 15%. Schulze and Zimmerman (2017) used a mixed integer programming model to optimize the objective function of maximizing the total material moved by a set of loader-trucks in an underground potash mine.

Simulation-based optimization was recently used by Ozdemir et al. (2019) to optimize a truck/shovel dispatching problem in an open-pit mine. The objective function of the optimization model was to maximize the total material moved in a shift. Wang et al. (2020) recently used a Genetic Algorithms Model to solve a model of the truck dispatching problem for an underground mine in China in which the objective function was to maximize production at the shift level (tonnes moved per shift) subject to constraints on the number of loading locations available, the capacity of the trucks, the material quantities available in each level, and the distance between loading levels. The results showed that the optimization model improved operational productivity by 8%.

Based on the literature review, we can identify the following trends: a) many researchers have shown that the use of a truck dispatching optimization model has improved the shift-level productivity in their mines; b) no researchers have (to our knowledge) formulated a goal programming model for this problem in underground gold mines. Hence, the research presented in this paper is an innovation on a problem of major economic consequence in underground gold mines.

3.3 Methods

The method in this chapter has three parts. First, the problem modeled is defined and illustrated with a conceptual figure. Second, the new mathematical formulation of the goal programming model for dispatching trucks in an underground gold mine is presented. Third, the case study, that portion of Red Lake's underground gold mine, which requires the dispatching of trucks, is described.

3.3.1 Definition of the Problem Modeled

Figure 3.1 (below) is a conceptual figure of the problem modeled, with a solution presented for one shift. First, observe that there are six levels. These represent different levels in the underground gold mine, and each level supplies gold ore of a different grade (in grams per tonne) and a fixed number of tonnes of ore are available per shift. Second, the elevator is the point of demand for gold ore. It carries gold ore to the surface where there is a target-demand in ounces of gold per day. The elevator also has a capacity constraint on the number of tonnes of gold ore it can move in one shift. Third, observe the distances between the six levels and the elevator. The distance and the slope of the path between each supply point and the elevator determines the transportation cost—all of which are different for each level. Fourth, observe that levels 1 and 3 have a shovel assigned. This assignment entails that, in the solution illustrated in Figure 3.1, levels 1 and 3 have been selected as supply points to meet the shift's demand. If a level has been selected, then it is assigned a shovel. There is a constraint on and cost for the number of shovels that can be used for any given shift. Finally, Figure 3.1 shows that, for each level selected, there is also a truck assigned. Trucks assigned to a level may be of different sizes and each size can move a fixed number of tonnes per trip.

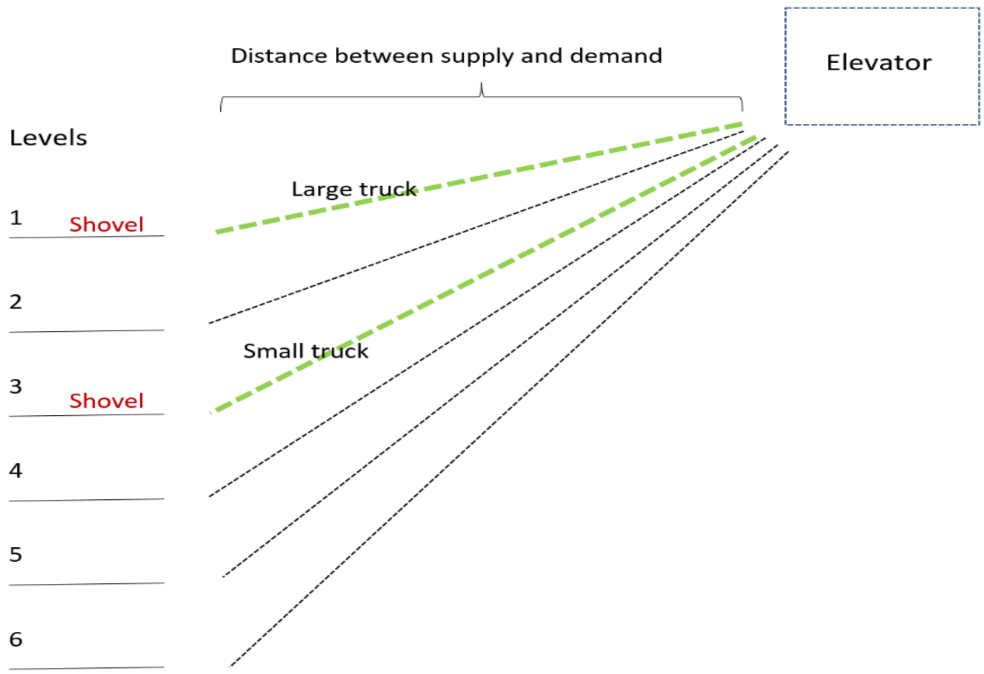

Figure 3.1: Conceptual figure of the problem modeled.

Hence, the problem to be solved is to assign a number of truck-trips (for each truck size) such that the following four goals can be met in a shift:

1. the gold goal (ounces per shift) be met.;
2. the goal for transportation cost ($) be met.;
3. the goal for the number of shovels used be met.; and
4. the goal for the number of trucks used be met.

Since these four goals can conflict with one another, and since the availability of trucks and shovels can fluctuate from shift to shift (owing to maintenance requirements on these machines), the problem is modeled as a goal programming model. The model is used not only to find the optimally satisfying solution for the decision-maker, but also to explore and quantify trade-offs to support the decision made.

3.3.2 Mathematical Formulation Model

The mathematical formulation of the goal programming model for dispatching trucks in an underground gold mine is presented below.

Indices and Sets:

i, I = *index and set of levels within the mine.*

j, J = *index and set of truck-types, by capacity.*

Parameters:

a_{ij} = *fraction of total shift time (C) required for one complete truck-trip assigned to level i using truck type j.*

b_j = *number of minutes required to load truck type j.*

C = *total number minutes in a shift.*

M = *arbitrarily large number.*

e_j = *capacity of truck type j (tons).*

D = *total demand for gold ore per shift at the elevator (tons).*

S_i = *supply of gold ore at level i, during the shift (tons).*

c_{ij} = *cost of trip needed for transporting one truckload of gold ore from level i using truck j.*

q_i = *grams per ton of gold ore at level i.*

G_ta = *goal value for transportation cost ($)*

G_tk = *goal value for number of trucks required.*

G_s = *goal value for number of shovels required.*

G_g = *goal value for mass of gold removed (grams)*

p_ta = *percent deviation factor for transportation goal variable = 1/ G_ta.*

p_tk = *percent deviation factor for truck goal variable = 1/ G_tk.*

p_s = *percent deviation factor for shovel goal variable = 1/ G_s.*

p_g = *percent deviation factor for transportation goal variable = 1/ G_g.*

w_ta = *penalty weight for transportation goal variable.*

w_tk = *penalty weight for truck goal variable.*

w_s = *penalty weight for shovel goal variable.*

w_g = *penalty weight for gold goal variable.*

Decision Variables

x_{ij} = *number of trips assigned to level i using truck type j.*

y_j = *total number of trucks of type j required.*

z_i = *1 if shovel at level i is used, 0 otherwise.*

s = *total number of shovels required in a shift.*

t = *total number of trucks required in a shift.*

Goal Variables:

g_ta^+, g_ta^- = *positive and negative deviations, respectively, from transportation goal ($).*

g_tk^+, g_tk^- = *positive and negative deviations, respectively, from truck goal (number).*

g_g^+, g_g^- = *positive and negative deviations, respectively, from gold goal (grams).*

g_s^+, g_s^- = *positive and negative deviations, respectively, from shovel goal (number).*

Objective Function

Minimize the total weighted percent deviations from all four goals.

$$(w\_ta * p\_ta * g\_ta^+) + (w\_tk * p\_tk * g\_tk^+) + (w\_s * p\_s * g\_s^+) + (w\_g * p\_g * g\_g^-) \qquad [1]$$

Subject to

The total number trucks required, of each type, is a function of the trucks assigned to all levels.

$$\sum_{i \in I} a_{ij}\, x_{ij} = y_j \qquad \text{for each } j \in J \qquad [2]$$

$$\sum_{j \in J} y_j = t \qquad [3]$$

If a level is assigned a truck, then it is also assigned a shovel.

$$\sum_{j \in J} x_{ij} \leq M\, z_i \qquad \text{for each } i \in I \qquad [4]$$

The total number of shovels required in a shift is the sum of all shovels assigned to all levels.

$$\sum_{i \in I} z_i \leq s \qquad [5]$$

There is a limit on the number of trucks that can be assigned to each level, based on the total time required to load all assigned trucks within the period of one shift.

$$\sum_{j \in J} b_j x_{ij} \leq C \quad \textit{for each } i \in I \qquad [6]$$

The total number of truck-trips is limited by the total demand per shift, in tonnes, at the elevator.

$$\sum_{i \in I} \sum_{j \in J} e_j x_{ij} \leq D \qquad [7]$$

The total number of truck-trips, assigned to each level, is limited by the total gold ore available at each level.

$$\sum_{j \in J} e_j x_{ij} \leq S_i \quad \textit{for each } i \in I \qquad [8]$$

The deviation from the goal in transportation-cost is a function of the total number of truck-trips assigned, the cost of each trip, and the chosen goal for transportation cost.

$$\sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + g\_ta^- - g\_ta^+ = G\_ta \qquad [9]$$

The deviation from the goal for the number of trucks assigned is based on the total trucks assigned, t.

$$t + g\_tk^- - g\_tk^+ = G\_tk \qquad [10]$$

The deviation from the goal for the number of shovels assigned is based on the total shovels assigned, s.

$$s + g\_s^- - g\_s^+ = G\_s \qquad [11]$$

The deviation from the goal for total gold removed is a function of the truck-trips assigned to each level and the grade at each level.

$$\sum_{i \in I} \sum_{j \in J} q_j x_{ij} + g\_g^- - g\_g^+ = G\_g \qquad [12]$$

Constraints on decision variables.

$x_{ij} \geq 0$ and integer [13]

$y_j \geq 0$ [14]

$z_i \in \{0, 1\}$ [15]

The objective function [1] is used to minimize the total weighted percent deviation from all goal variables. By default, all weights are valued at 1, unless otherwise stated. Equation [2] defines the number of trucks required per shift, for each truck-type, based on the assignment of trucks to levels. The total number of trucks required, t, is defined in Equation [3]. Equation [4] defines whether or not a shovel is used at a given level. Since the use of a shovel (z_i) is triggered by the dispatching of a truck to that level (x_{ij}), the variable representing the use of a shovel (z_i) must be binary for this equation to work--see Equation [15]. Equation [5] defines the total number of shovels used in a shift. Equation [6] limits the maximum number of shovels required at each level to be 1. This constraint is based on the reasoning that the total number of minutes that a shovel may be used in loading trucks may not be more than the number of minutes in a shift. Equation [7] limits the total ore removed during the shift from exceeding the total demand for the shift. Equation [8] limits the ore removed by dispatched trucks, of varying capacities, from exceeding the supply of ore at each level. Equation [9] defines the goal variables for transportation. Each trip dispatched is a round-trip, from the demand point (the elevator) to the supply-point, the shovel at a given level. Of key importance here is the parameter c_{ij}, which varies for each level, depending on the distance travelled and slope at which a truck is required to travel, both empty and full. Equations [10] and [11] define the goal variables for trucks and shovels. Equation [12] defines the goal variables for gold. It should be noted that the goal for gold (by historical convention) is in ounces, and that this goal is based on the estimated grams of gold per ton of gold ore, which varies from level to level. Equation [13] ensures that the number of trips dispatched to each level is integer. Equation [14] constrains the number of each truck type required to be non-negative. The variable, y_j , for the work in this chapter, is not constrained to be integer. This is because an integer constraint required excessive computing time and the variable only needed to be rounded up in order to interpret the number of trucks of each type required by the dispatching solution. Equation [15] ensures that the variable representing whether a shovel is used at a given level or not be binary.

3.3.3 Case Study

The underground gold mine in Red Lake, Ontario is greater than 50 years. Fifty years of continuous underground mining operations have history resulted in 52 levels reaching a depth of up to 2.4 km. Figure 3.2 (below) illustrates that the first 38 levels are connected to a main shaft into which mined material is dumped. At the bottom of level 38 is an elevator which carries the mined material to the surface. Figure 3.2 also illustrates that, below level 38, there are 14 levels which are not connected to the main shaft. Material mined from these levels (levels 39 to 52) must be transported by trucks to the elevator at level 38. In general, the deeper the location of each level, e more costly is the transportation required to service it. The problem in this case study is the dispatching of trucks, per shift, to these 14 levels such that the multiple objectives are optimally satisfied.

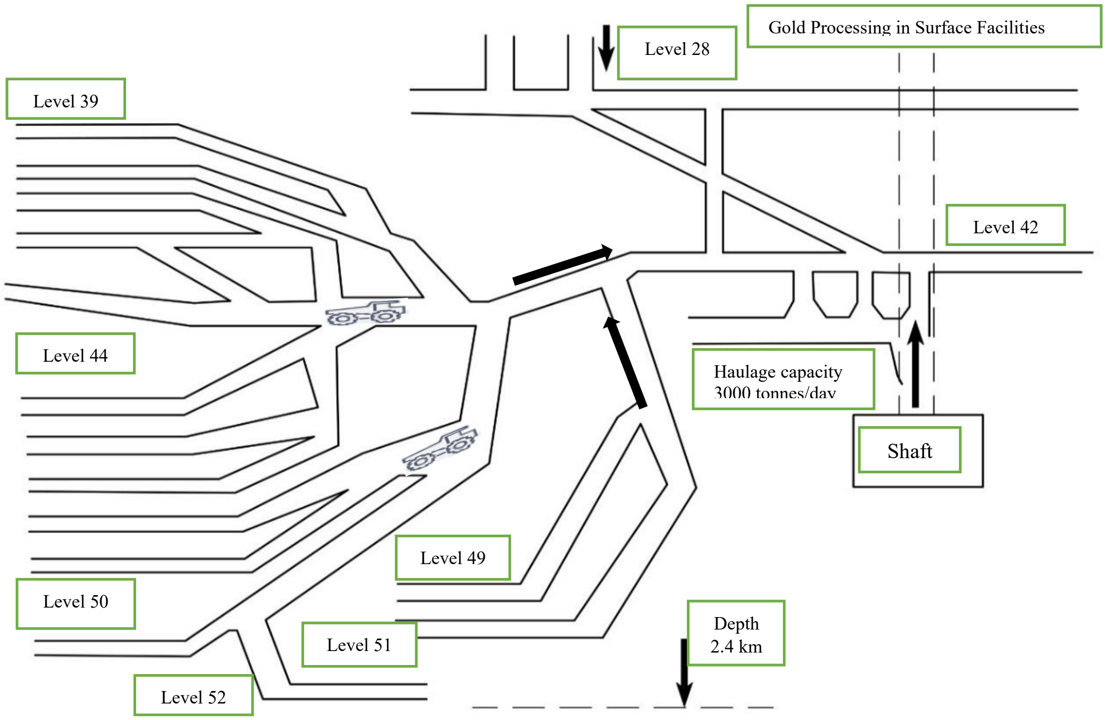

Figure 3.2: Underground transportation network for trucks at Red Lake Mine (RLGM).

At present, there is no optimization model used for dispatching trucks at the Red Lake mine. Decisions are supported using analysis of data on spreadsheets. The managers of Red Lake mine expressed interest in the development of an optimal dispatching model because of the cost of trucking materials in the underground mine is a major one. The managers wanted a model that addresses several objectives. First, they wanted a model that optimizes the movement of gold (measured in ounces) for each scheduled 12-hour shift. Second, the decision-makers at Red Lake wanted to minimize their transportations costs. Third, the decision-makers wanted to a model to support their decision on what size of truck should be assigned to move material from each level. Finally, since the assignment of each level for loading a truck also requires a shovel and an operator, the managers also wished to use a model to explore costs *versus* benefits of reducing the assignment of shovels for each shift. Given these four objectives the analysts and decision-makers at Red Lake wanted a model that could enlighten decisions on the trade-offs involved between competing objectives. For these reasons, a goal programming model was, therefore formulated and evaluated to address these concerns at the Red Lake mine.

3.4 Results

Since the decision-makers at the Red Lake mine were interested in quantifying the trade-offs among the competing objectives in this problem, a pre-emptive method was used in applying the goal programming model.   In the pre-emptive method (Eschenbach et al. 2001), goals are ordered according to priorities, and the values assigned to each goal are determined by executing a sequence of scenarios. For example, in the results shown in Table 3.1 (below), the first scenario was run with gold as the top priority. Gold, therefore, was the *only goal* used in the model's objective function in scenario 1. The achieved value for gold, in scenario 1, was then used as gold's goal-value in scenario 2. This sequential method was used for directing the assignment of all goal values.

The priorities underlying the pre-emptive method were selected in consultation with the decision-makers at Red-Lake. The priorities of the objectives were ranked as follows:

1. Gold removed
2. Transport cost
3. Shovels used
4. Trucks used.

The results of the four scenarios are shown in Table 3.1 (Note: the values in square brackets are *achieved* values of goals that were not optimized in the objective function but resulted from the optimal solution).

Table 3.1: Results for four scenarios using pre-emptive method.

Scenario	Goals in Objective Function	**Goal Values**				Achieved Values			
		Gold (g.)	**Transport ($)**	**#Shovels**	**#Trucks**	Gold (g.)	Transport ($)	#Shovels	#Trucks
1	Gold	**10,000**				7,570	[5,848]	[10]	[11]
2	Gold + Transportation	**7,570**	**5,000**			7,553	4,984	[11]	[8]
3	Gold + Transportation + Shovels	**7,570**	**5,000**	**9**		7,509	5,007	9	[6]
4	Gold + Transportation + Shovels + Trucks	**7,570**	**5,000**	**9**	**4**	7,425	5,024	9	4

The results in Table 3.1 yield several noteworthy observations. First, in scenario 2, one can observe the trade-off between gold removed and transportation costs by comparing the achieved values for these goals in scenarios 1 and 2. Here we observe that, by adding transportation cost as a goal in scenario 2, transportation costs were reduced from $5,848 per shift to $4,984 — a reduction of 14.8% — achieved by lowering the total gold removed by less than 1% (from 7,563 g to 7,553 g). The improved solution of scenario 2 shows the benefit of dispatching trucks for both gold and transportation costs simultaneously using this model.

Second, scenario 2 also shows that the reduction in transportation costs resulted in an increase in the number of shovels used (from 10 to 11). Why did this happen? By comparing the solution of scenario 1 with scenario 2 (in Table 3.2, below) we observe two things. First, that when the goal was only for gold, the solution was easy to form — simply send the smaller trucks to the levels with the richest deposits, regardless of cost. Smaller trucks were sent because carrying smaller discrete volumes of ore is makes it easier to remove, as closely as possible, the total discrete volume of ore supplied at the ore-rich levels than if one dispatched a discrete set of larger trucks. Second, Table 3.2 also shows that different levels (in scenario 1 versus 2) were accessed in order to reduce transportation costs. Recalling that the depth of a level influences its transportation cost, we can observe that scenario 2 added the less costly levels 44 and 46 and removed the more costly level 50. Hence, scenario 2 showed an unintended consequence of adding the objective to reduce transportation costs; namely, that in order to reduce transportation costs and to meet the gold goal, an extra level was added to the solution, requiring an extra shovel. This unintended consequence shows the need for adding shovels as an objective in a goal programming model of this dispatching problem.

Table 3.2: Solutions for four scenarios. (Note: the values in the scenario columns represent the number of trips dispatched to each level, for each truck type).

Level	Truck Capacity (tons)	Scenario			
		1	2	3	4
39	17	5	5	5	1
40	17	5	1	5	1
41	17	6	1	5	1
42	17	4	0	2	0
43	17	4	1	1	0
44	17	0	1	0	3
45	17	5	0	2	2
46	17	0	1	0	0
47	17	4	0	0	3
48	17	0	0	0	0
49	17	0	0	0	0
50	17	1	0	0	0
51	17	3	1	0	0
52	17	4	0	0	0
39	30	0	0	0	2
40	30	0	2	0	2
41	30	0	3	0	3
42	30	0	2	1	0
43	30	0	1	1	2
44	30	0	0	2	1
45	30	0	3	2	2
46	30	0	0	1	0
47	30	0	2	2	1
48	30	0	0	0	0
49	30	0	0	0	0
50	30	0	0	0	0
51	30	0	2	1	2
52	30	0	2	2	2

Thirdly, scenario 3 shows that, by adding an objective of 9 shovels to the model's objective function, we were able to meet this objective and improve upon the solution in scenario 2, which entailed 11 shovels — a reduction in shovel cost of 18.2%. This improvement came with a small trade-off: a reduction in gold removed in scenario 3 versus scenario 2 (less than 0.1%) and a slight increase in transportation cost (less than 0.1%). Table 3.2 also shows that the solution for scenario 3 is radically different from that of scenario 2. These results show that, with a slight trade-off for two objectives, it is possible to achieve a major improvement in a third objective. Hence, the solution to scenario 3 illustrates well how the dispatching problem is suitable for multiple objective optimization through goal programming.

Finally, scenario 4 (see Table 3.1) shows that by adding a fourth objective, i.e., the number of trucks required, the overall solution was further refined. Comparing scenarios 3 and 4, we observe that the number of trucks was reduced from 6 to 4 (33.3% reduction). This came with a trade-off of reducing the gold removed by 1% and of increasing the transportation cost by less than 1%. The number of shovels used remained the same. Table 3.2 shows that the solutions of scenarios 3 and 4 differ in a predictable manner; namely, the number of trips assigned to the larger capacity trucks was greatly increased in scenarios 4 in order to meet the targets with fewer trucks. The use of this model for addressing the problem of selecting the optimal number and size of trucks could, of course, be further refined; but scenario 4 illustrates its capacity to engage in the exploration of meaningful trade-offs.

The results of this chapter also incudes a trade-off analysis of of competing objectives. Figure 3.2 (below) illustrates the trade-off between transportation costs and total gold removed in a shift.

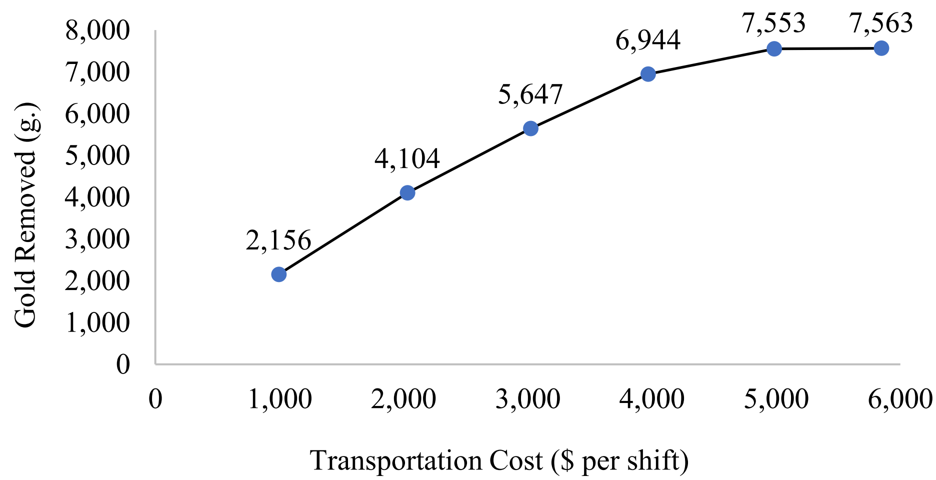

Figure 3.3: Solutions showing trade-off between transportation costs and gold removed.

The results in Figure 3.3 show that the increased returns in gold resulting from increased expenditures on transportation begin when transportation costs reach $4,000 per shift and that any spending beyond $5,000 yields very little improvements in terms of gold removed. They also illustrate the cost of reducing the budget on transportation costs per shift in terms of reduced gold. Figure 3.3 shows the effect on gold removed when assigning different numbers of trucks to the problem instance

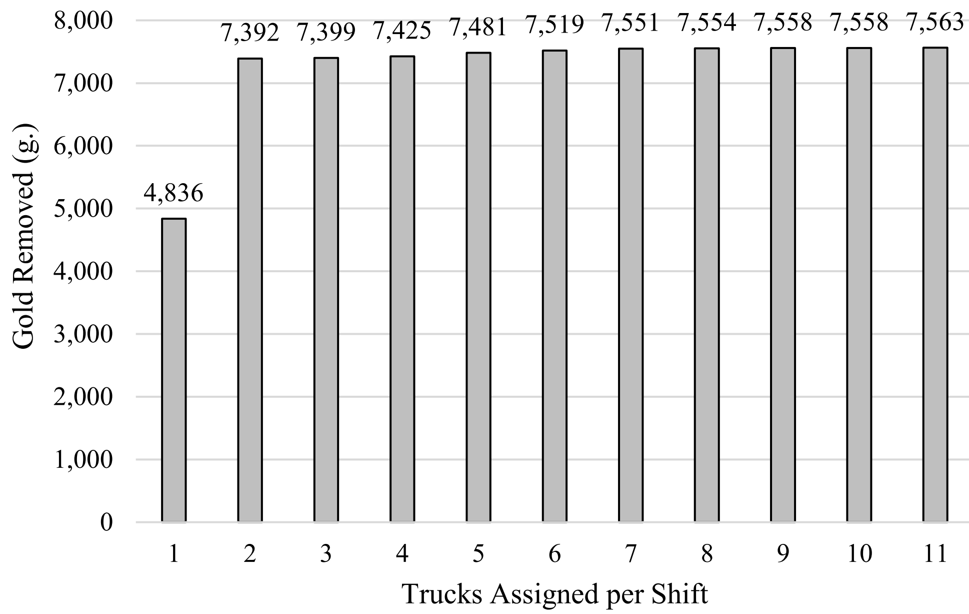

Figure 3.4: Solutions showing trade-off between trucks assigned and gold removed.

Figure 3.4 shows the effect on gold removed when assigning different numbers of trucks to the problem instance. It is interesting to note that diminishing returns set in rapidly after two trucks, and that small gains in gold removed persists up to the maximum amount of gold removable, requiring 11 trucks (same as scenario 1). This is possible for the reasons given above; namely, that in the solution with 11 small (17-ton) trucks. Smaller trucks were dispatched because carrying discrete volumes of ore is easier to remove, as closely as possible, the total volume of ore supplied at rich levels than if one dispatched a discrete set of larger trucks.

3.5 Discussion

The results show that the goal programming model presented in this chapter may be useful to decision-makers in two respects. Firstly, the model is (i) well-suited to the problem; and secondly, the model is (ii) feasible for operational planning. Each of these model attributes are evaluated below.

In the classic formulation of the optimization model of the truck dispatching problem in mines, trucks are dispatched to shovels in such a way that the productivity of trucks and shovels is maximized while maintaining target grades (Elbrond, et al. 1987). The new goal programming model introduced in this work does not essentially depart from this representation of the problem, but rather adds another quality to the solution: it provides the best satisficing solution under a varying amount of resources and priorities of the goals. For example, instead of "locking" the gold target as a constraint, as the classic formulation entails, the goal programming model allows one to find an optimal, feasible quantity of removable gold given a set of other objectives. Table 3.1 shows how the optimal achievable level of removable gold changes as other objectives (in scenarios 1-4) are added to the problem. In other words, rather than set a hard constraint, *a priori*, on the economically most important objective of the problem (i.e., gold), the goal programming model finds an optimal feasible value for removable gold. Therefore, the long-term economic benefit of dispatching trucks to shovels with such a high objective of the problem optimized is worthy of consideration. In addition, the goal programming model provides the decision-maker with insight into trade-offs and diminishing returns, thus allowing a deeper insight and confidence in the desirability of the solution selected.

Second, the model presented in this work is suitable to the practice of daily operational planning for several reasons. 1) The availability of resources, such as trucks or shovels, may fluctuate due to breakdowns and consequent repair times. The consequences of fluctuating resource availability can be quantified directly in terms of lost gold production, thereby adding the proportionate degree of urgency to allocating resources for repair. 2) The model itself is quite easy to build and solve in Microsoft Excel (the model was solved in Excel in in less than two minutes of execution-time on an Intel Core i7). For this reason, it can be easily adopted by an analyst working for a mining firm and re-run in the middle of a shift should any equipment failure require a new dispatching solution. In this way, stochastic events inherent the dispatching problem can be addressed in this by using this model. Such an opportunity can facilitate improved decision-making, with minimal effort, especially for underground mining firms that are not using optimization software for the dispatching problem. Unlike the above-ground dispatching problem, for which commercial software optimization models exist, the underground mining dispatching problem does not have a selection of commercially-available software packages to choose from. The practical implementation and execution of this model on Excel therefore makes the development of this model truly practicable for underground mine managers in the gold industry.

3.6 Conclusion

In this chapter, a new formulation of the truck dispatching model for an underground gold mine has been presented. The model was formulated as a goal programming model and applied to Red Lake's gold mine in Ontario, Canada. The results showed that major reductions in transportation costs, shovels used, and trucks required can be achieved with a minimal decrease (less than 0.1%) in the maximum quantity of gold that can removed in a shift. Given the scale of these reduced costs, this model will be a valuable addition to the decision-makers seeking to increase the efficiency of their dispatching operations in an underground gold mine.

Future research on the problem of modeling this dispatching problem would be in exploring the applicability of a goal programming as a useful approach for modeling the dispatching problem in different types of underground mines; i.e., to determine whether its benefits can be expanded to mines other than gold mines.

Chapter 4

A Network Flow Model of Ore and Waste for Daily Operations in an Underground Gold Mine

4.1 Introduction

In underground mines, the problem of scheduling the weekly locations of mining operations has a major impact on the productivity of the entire mining operation (Yun 1990). The objective of the weekly schedule is to maximize production within the constraints of the mine's strategic and tactical plans. The weekly schedule is difficult to optimize for two reasons. First, the solution (i.e., the optimal locations selected to mine over the next weekly period) is constrained by the feasibility of transporting the mined material from the selected locations, through the capacity-constrained transportation network, to the surface. The transportation network in an underground mine has strict capacity constraints on the mass of material that can move, per unit time, through many links within its transportation network. The second reason this problem is difficult to solve optimally is that commercial software is currently not available for such a planning problem in underground gold mines. There is no general-purpose software package that has been designed to represent realistically the diversity of constraints peculiar to the many underground mine types-- including underground gold mines. As a result, planners are forced to resort to using 'rules of thumb' to solve this problem.

In underground gold mines in particular, the operational planning problem of scheduling the weekly locations of mining operations, subject to transportation constraints, has, to our knowledge, not been formulated as an optimization model at the operational scale of planning. The problem has therefore been solved using ‘rules of thumb’ (i.e., heuristics) typically executed on a spreadsheet. Finding an optimal rather than a heuristic solution to this problem is important for two reasons. First, the difference between an optimal and a heuristic solution may have major differences in the objective function value of the problem’s solution; i.e., the mine’s operational productivity, measured in ounces of gold delivered to the surface per day. This is because, in a gold mine, the grade of the gold-ore (measured in g/tonne) varies from location to location. In other words, the value of the locations selected for mining can vary greatly because of the variance of grade across space, in an underground gold mine. Hence, if this were, for example, a coal mine (e.g., Brzychczy 2014), the difference between the solutions generated using “rule of thumb” method *versus* an optimization model may not be great, because the value of the grades at each selected location differ less significantly in a coal mine than in a gold mine. Since this is a gold mine, the value of the optimal feasible solution is highly sensitive to slight differences in the locations selected for mining. Therefore, the difference between an optimal and a heuristic solution to this problem may be quite important economically. A second reason for the importance of using an optimization model on this problem is that the above-ground processing facilities are constrained, in their daily productivity, by the value of the gold-ore that is delivered to the surface each day. Since the value of the gold ore delivered to the surface each day is the objective function of this operational problem’s optimization model, the solution to this problem has a direct impact on the productivity of the above-ground processing facilities. In other words, the solution to this problem, when seen in a broader context, can be seen to act as a constraint on the economic productivity of the entire mining facility itself.

The objective of this chapter is to formulate and evaluate a new operational planning model for the underground gold mining problem of scheduling the weekly and optimal location of mining operations, subject to transportation constraints on the flow of material. The formulation will be of a mixed-integer, network-flow model. This model will be applied to a case study at the Red Lake Gold Mine in Ontario, Canada. The model will be evaluated by comparing its solution to that of a greedy heuristic currently used. In this way, we will evaluate a larger question: whether the benefit of using a specific operational planning model, for constraints peculiar to underground gold mine, is of any economic consequence.

The outline of this work is as follows: first, a Literature Review is presented; second, the formulation of the new model and the heuristic algorithm are presented in the Methods; third, the Case Study, Red Lake Gold Mine, is described; fourth, the Results are presented, in which the optimal solution is compared to the heuristic solution; and finally, a Discussion of the paper's results and their significance is evaluated.

4.2 Literature Review

Literature reviews on the use of operations research in mining in general (e.g., Newman et al. 2010; Bjorndall et al. 2011; Kozan and Liu, 2011), and underground mining in particular (Chowdu et al. 2022), indicate that: while there is a wealth of research in the development of optimization models for strategic and tactical problems in mining, published work on scheduling models used at the operational scale have been much more scarce. The literature to be reviewed on this problem is therefore brief and shows a great diversity of models formulated for operational production planning. For this reason, the review is presented chronologically, not thematically.

Nehring, et al. (2010) developed a model for a sub-level stoping mine, which was used to solve the problem of scheduling and allocating machines for the transportation of extracted ore from the draw-points, via intermediate storage, to a haulage shaft. The model also included decisions on a second-stage movement of ore; i.e., transporting ore from an ore-pass to a crusher. The problem was formulated as a mixed integer programming (MIP) model to allocate machines to different draw-points, on a shift basis, over a period of 2 months. The objective function of the model was to minimize the deviation from production targets subject to constraints on machines and crew. The model's solutions were evaluated on a simulated mine.

Martinez and Newman (2010) developed a comprehensive operational scheduling and allocation model for an iron ore mine in Northern Sweden. The model minimized deviations from monthly production targets subject to a host of operational constraints, many of which were peculiar to sub-level caving. The MIP model was solved on a real-world dataset, using a heuristic algorithm, to within 5% of production targets.

Howes and Forrest (2012) described an approach to improving operational decision-making at a mine in Bulgaria. A key strategy introduced in this work was short interval control. This involves the use of real-time production information to provide a central monitoring and control room with the real-time status of all tasks in the mine. This comprehensive communications infrastructure was designed to support key frontline decision-making on operational resource allocation achieves the maximum efficiency for each shift. At present, the decisions are made by management in the central control room, but the development of operations research models to support management decisions in this environment is the next step in the evolution of this ambitious project, and therefore a fruitful field of future research on operational scheduling in mining.

Nehring et al. (2012) addressed the task of integrating short- and medium-term production plans. Their method was to combine the short-term objective of minimizing deviation from targeted mill feed grade with the medium-term objective of maximizing net present value (NPV) into a single mathematical optimization model. Their short-term problem was not constrained by capacities on the transportation network but. Their resulting solution was a global optimum of the two planning problems.

Little, Knights, and Topal (2013) evaluated the advantage of simultaneously integrating decisions on both stope layout and production scheduling into one model. They found that the solutions generated by the integrated model were superior to those using different models sequentially. The benefits of integrating separate but interdependent models, as demonstrated by these authors, are promising.

Schulze et al. (2016) scheduled a mobile production fleet in an underground, room-and-pillar, potash mine. The objective of the model was to minimize the make-span, i.e., to create the shortest logical project schedule, by efficiently using project resources and adding the lowest number of additional resources to each sub-task. The problem was formulated as an MIP model and solved using a commercial solver. The authors continued to explore the room and pillar-scheduling problem by developing a heuristic solution method in Schulze and Zimmermann (2017).

Campeau and Gamache (2019) presented an optimization model for short-term scheduling of excavation, hauling, and backfilling activities at a cut-and-fill gold mine in Canada. The objective function was to maximize total discounted tonnage extracted over an eighteen week planning horizon, subject to resource and sequencing constraints. The authors observe that the real value of their solutions rests heavily on the quality of the tactical plan's selected sequence of blocks, on which their model acts.

Manriquez et al. (2020) developed a simulation-optimization model to generate short-term production schedules for improving the schedule adherence using an iterative approach. In each iteration of this framework, a short-term schedule was generated using a mixed-integer linear programming model that is simulated later using a discrete-event simulation model. The model was not subject to capacity constraints on the transportation network.

From this review of the literature, one can draw two observations. First, that the problems modeled for operational planning in underground mines are not generic but quite diverse and specific to mine types. The models formulated were often custom-built for the particular extraction method of the mine and its design. It is perhaps for this reason that there exists no commercial optimization software that is universally employable for operational scheduling in all underground mines, as there is for tactical planning of underground mines (Newman 2010). A second observation that can be drawn from the literature review is that the particular problem addressed in this paper (i.e., the optimal operational-scale scheduling of gold ore and waste flow, in an underground gold mine) has not been addressed within prior research.

4.3 Methods

The Method is divided into 4 parts. First, a description of the problem, with a conceptual figure, is given; second, the mathematical formulation of the optimization model is presented; third, the heuristic algorithm used in this paper, to represent the current decision-making procedure at the mine, is given. Finally, the case study and data used are described.

4.3.1 Description of Modeled Problem:

A conceptual figure of the problem is presented in Figure 4.1. Here we observe a simplified representation of an underground gold mine. First, observe that there are 9 levels. Each level may be regarded as a source-node, having a different: i) grade of ore (g/tonne); ii) mass of ore that may be removed daily (tonnes/day); and iii) mass of waste that must be removed if ore is removed (tonnes/day). These are the network's source nodes. Second, observe that there are two types of shafts for downward movement of mined material: ore and waste shafts.

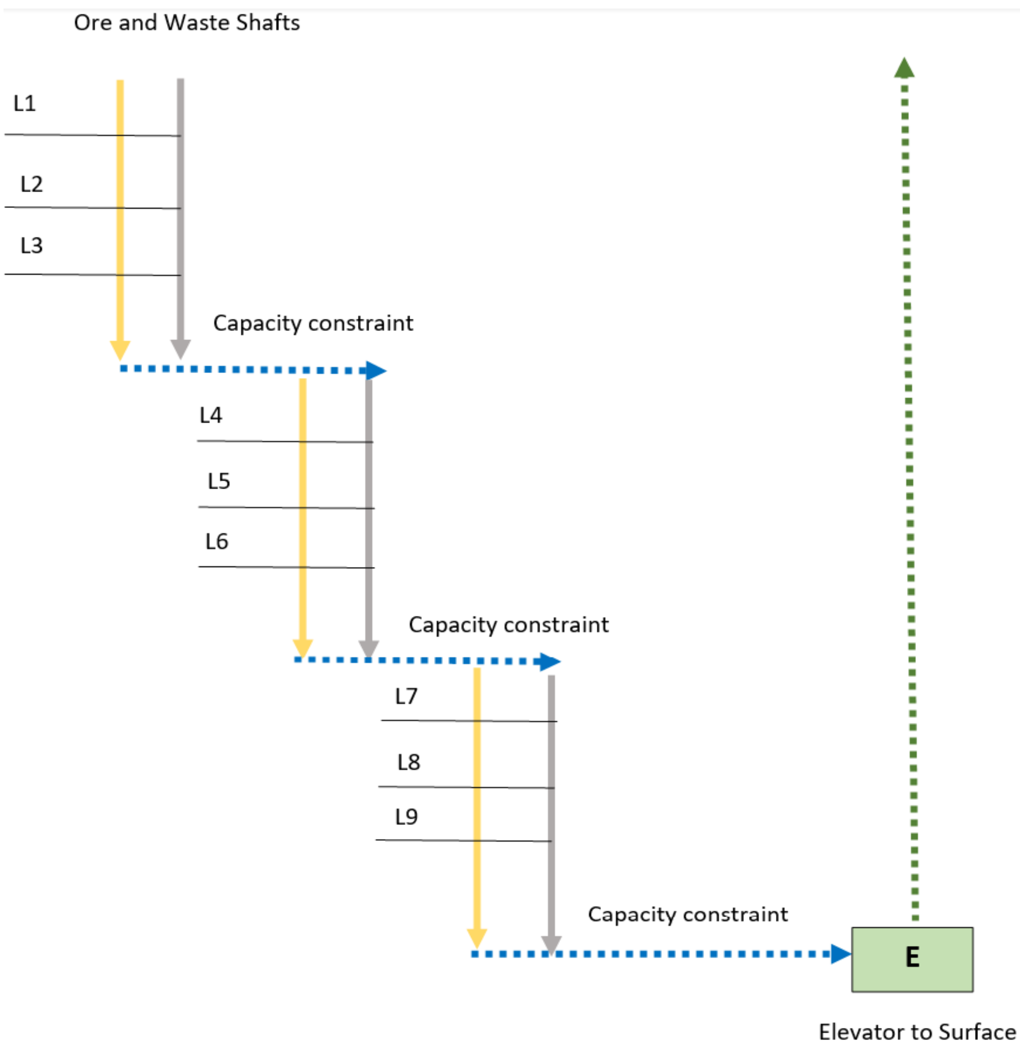

Figure 4.1: Conceptual figure of the modeled problem.

There are no capacity constraints on these shafts. Third, observe that, at the bottom each shaft, horizontal transportation of both materials occurs. This horizontal transportation has a daily capacity constraint. The shafts and horizontal transportation arcs are the network's trans-shipment arcs. Fourth, observe that, at the end of the network, there is a capacity-constrained elevator to the surface. This is the network's terminal node where there is a daily demand for ore and a daily demand for the mass of waste resulting from the mining of the ore. Hence, the problem may be summarized as: select a set of levels for daily operation such that the mass of gold removed (in ounces) is maximized, subject to: i) capacity constraints on ore transportation (tonnes per shift); ii) daily ore targets at the above-ground processing facility (in tonnes) is met; and iii) the waste material accompanying ore removal is removed. The problem is formulated as a network flow model where each level is a supply node and the elevator is the demand node. Constraints on the flow of material occur at transition nodes. Unless these transportation capacity constraints are used in planning, the movement of both ore and waste through the mine could be stopped during a shift because a transportation corridor may become backed-up from crews trying to move too much ore through a corridor, with too little capacity for such a quantity, within the planned period. In addition, the selection of a level must be represented by a binary decision variable because, if real numbers were used, a solution with tiny, fractional mining of levels could occur. This is not feasible in practice because the fractions might be very small and therefore a solution could be produced where it is not worth sending machines and a crew to mine a level with a tiny, fractional amount of ore. Finally, the daily schedule of production is to be found for a planning horizon of 7 days.

This problem is difficult to solve, for it is a combinatorial optimization problem that may be reduced to the famous knapsack problem. In the knapsack problem, one is given a set of items, each with a weight and a value, and one must select a set of items to include in a knapsack such that; i) the total weight is less than or equal to a given limit; and ii) the total value of the contents of the knapsack is as large as possible (Salkin and Kluyver 1975). The operational mining problem described above can be reduced, in terms of it computational complexity. This is because, apart from the transportation constraints, the problem is the same: namely, selecting a set of levels (i.e., items) so that the total mass of ore selected is less than or equal to the limit (i.e., upper bound) set by the elevator's capacity and the objective is that the total value of the gold (i.e., value of selected items) from the levels selected discretely for mining be as valuable as possible. The reduction of this problem to the knapsack problem also implies that, in its computational complexity, the problem's model is NP-hard, as is the knapsack problem (Salkin and Kluyver 1975).

4.3.2 Formulation of the New Model:

The mathematical formulation of the model is presented below.

Indices and Sets

n, N = *index and set of levels in the mine.*

t, T = *index and set of planning periods.*

m, M = *index and set of materials moved through the network (i.e., ore or waste material).*

i, j, J = *index and set of nodes in the network.*

B = *set of intermediate (transshipment) nodes.*

C = *set of arcs with capacity constraints.*

D_{ij} = *capacity on arc i-j (tonnes per day).*

O_n = *set of arcs flowing out from node on level n.*

I_n = *set of arcs flowing into node on level n.*

F = *the set of arcs flowing into the* network's terminal node *(a set of 1).*

Parameters

s_{mn} = *mass of material type, m, available for removal at level n of the mine (tonnes).*

g_n = *estimated mass of gold ore available at level n of the mine (grams).*

Decision Variables

y_{nt} = *1 if material is removed from level n in period t, 0 otherwise.*

x_{ijmt} = *flow of material, m, through arc i-j, in period t (tonnes).*

Objective Function:

Maximize total mass of gold removed (in grams) over all periods

$$\sum_{n \in N} \sum_{t \in T} y_{nt} \, g_n \qquad [1]$$

Subject to:

Mine each level not more than once.

$$\sum_{t \in T} y_{nt} \leq 1 \qquad \text{for each } n \in N \qquad [2]$$

If a level is mined, it is a source of ore and waste material flow.

$$\sum_{(i,j) \in O_n} x_{ijmt} - \sum_{(i,j) \in I_n} x_{ijmt} = y_{nt} \, s_{mn} \qquad \text{for each } n \in N,\ m \in M,\ t \in T \qquad [3]$$

- *Transition nodes defined.*

$$\sum_{(i,j) \in O_n} x_{ijmt} - \sum_{(i,j) \in I_n} x_{ijmt} = 0 \qquad \text{for each } n \in N,\ m \in M,\ t \in T \qquad [4]$$

- *Terminal node defined.*

$$\sum_{(i,j) \in F} x_{ijmt} - \sum_{(i,j) \in F} x_{ijmt} = \sum_{n \in N} y_{nt} \, s_{mn} \qquad \text{for each } m \in M,\ t \in T \qquad [5]$$

- *Capacity constraints.*

$$\sum_{m \in M} x_{ijmt} \leq D_{ij} \qquad \textit{for each } ij \in C,\ t \in T \qquad [6]$$

Binary and non-negativity constraints.

$$y_{nt} \in \{0,1\} \qquad \textit{for each } n \in N,\ t \in T \qquad [7]$$

$$x_{ijmt} \geq 0 \qquad \textit{for each } ij \in J,\ m \in M,\ t \in T \qquad [8]$$

The objective function [1] of the model is to maximize the total mass of gold removed (in ounces) during the daily shift. The mass of gold is based on the tactical plan's estimated grade of each block at each level (measured in grams per tonne) and the total mass of gold ore and waste (measured in tonnes) that is currently available to be mined at a given level. The first constraint [2] ensures that no level may be mined more than once over the planning horizon. The second constraint [3] defines the potential sources of flow through the network. This constraint ensures that, if a given level, n, is mined in period t, then each material type, m, will flow out of the node on this level and into the network. Note that the flow of each material type (ore and waste) is tracked separately from each source. Constraint [4] is a standard flow balance equation for transition nodes in a network model. Equation [5] defines the terminal node and the mass of each material type demanded at the terminal node. Note that the total mass of each material type refers to the total mass of each material type that was mined during each period. There is an upper bound on this value implicit in the capacity constraint on the arc connected to the terminal node. Equation [6] defines the capacity constraints on the flow of material types imposed on the set of arcs with capacity constraints. Equation [7] ensures that the decision variable, y_{nt}, is binary. This variable is binary for two reasons. First, the mass of material removed from each level must be discrete; otherwise, the model might produce solutions that are operationally infeasible (e.g., tiny masses of material to be scheduled for removal from a level). Second, the binary decision variable is needed to trigger the flow in equation [3]. Equation [8] ensures that a negative flow value is not possible.

The above model was solved, in this study, using the branch and bound algorithm of Gurobi. ©

4.3.3 Description of Heuristic Algorithm:

Given the absence of an optimization model to solve this operational problem, our industrial partner had been using a heuristic method (i.e., rules of thumb). This method will now be described, for its results will be compared with the results of the new optimization model in order to evaluate the latter.

Given that the objective function of the model is maximize the gold ounces removed over the planning horizon, subject to capacity constraints, a greedy search was used. A greedy search heuristic has been used quite successfully on many versions of the knapsack model (Ackay et al. 2007). In this greedy search, the levels were sorted from highest to lowest grade, and selection proceeds from highest to lowest, subject to whether the addition of a level to the schedule violates the transportation capacity constraints of in the mine (see equation [6] above). An algorithmic flowchart of the greedy search is presented in Figure 4.2 (below). Here, the search is repeated at the start each of the 7 days, and all candidate levels are rendered eligible for inclusion in the schedule at the start of each day.

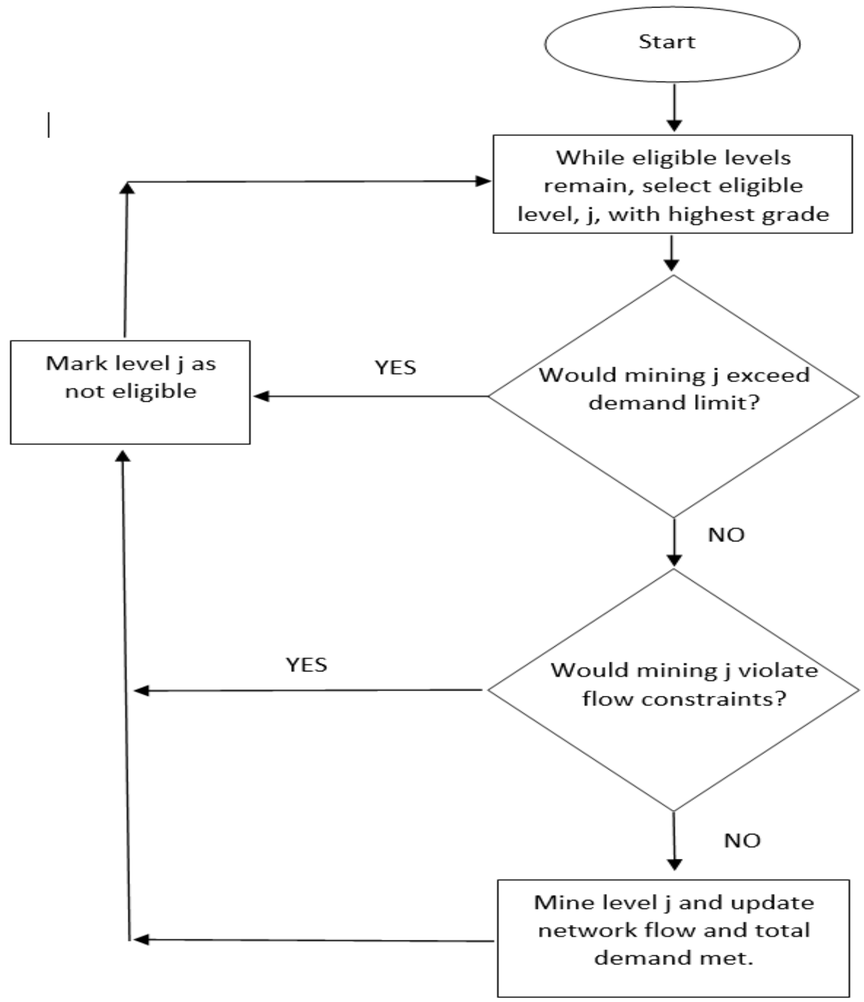

Figure 4.2: Algorithmic flowchart of greedy heuristic.

In the greedy heuristic described above, there is a demand limit on two types of materials mined: gold ore and waste. In addition, a second heuristic is used in the paper, called heuristic 2. Heuristic 2 places a demand limit only on the ore and allows the mass of waste to exceed its target. Heuristic 2, therefore, allows for greater opportunity to maximize the value of the objective function while running the risk of mining a slight excess of waste. Heuristic 2 is sometimes used by the planners at Red Lake. Hence, the trade-off involved in using heuristic 1 versus heuristic 2 is a practical one for decision makers to explore and use. In this paper, both heuristic 1 and heuristic 2 are used and their results are presented and compared with the results of the new optimization model.

4.3.4 Case Study

The case study is the Red Lake underground gold mine, located in Red Lake, Ontario, Canada. The mine is approximately 50 years old and is currently under the management of Newmont-Goldcorp Corporation, our industrial partner. The levels of production are shown in Figure 4.3 (below). Each level contains discrete masses of both ore and waste material. Based on data shared with us by our industrial partner, the mass of each block, in each level ranges from 3,500- 5,000 tonnes. Each block has been scheduled, in the tactical plan, for mining within the calendar-year. The objective of this model is to transform the annual tactical plan into an optimal weekly operational plan of production. There are 19 levels currently eligible for production, based on the tactical plan.

The levels scheduled for tactical operation at the Red Lake gold mine are presented in Figure 4.3 (below). First, observe that there are three sets of levels. Second, observe that, at the bottom of the ore and waste shafts of each level, horizontal transportation of material is required, and there is a daily haulage capacity constraint on this. Third, observe that, at the bottom of the mine, there is an elevator to the surface with a daily capacity of 3,000 tonnes/day. The daily demand at the surface is for 2,000 tonnes of gold ore and 1,000 tonnes of waste to be sent to the surface daily.

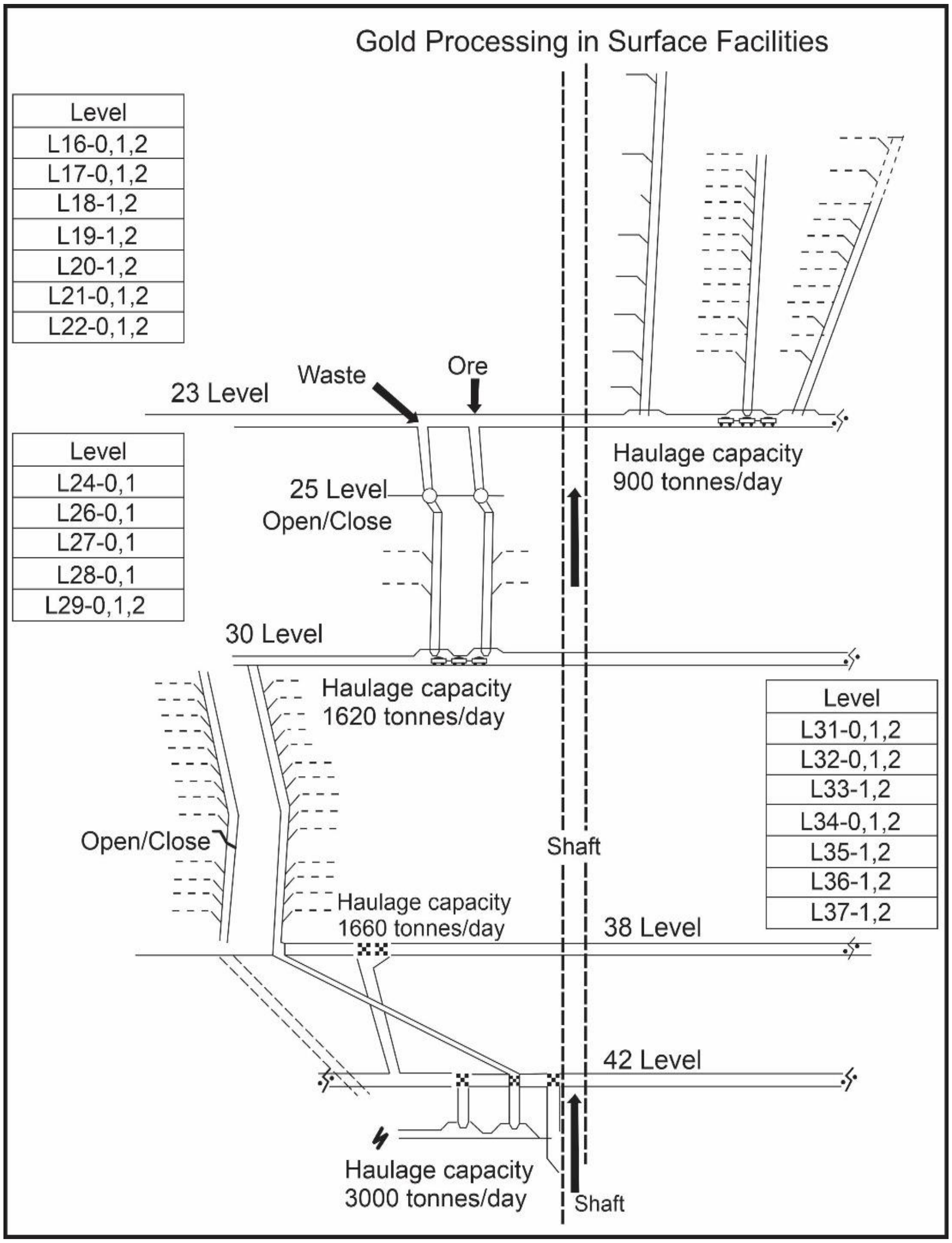

Figure 4.3: Ore and waste flow in underground mine at Red Lake, Ontario, Canada.

Each block at each level at Red Lake differs by: i) the grade of the ore (g/tonne); ii) the mass of ore that can be removed in one day (tonnes/day); and iii) the mass of waste that must be removed in one day (tonnes/day), if ore is removed. The values for these parameters are presented in Table 4.1. It should be noted that parameters in Table 4.1 for the grade of ore are not real. Our industrial partner, understandably, wished to keep these values on grade private. The parameters for the grade of ore were therefore generated using a random number generator such that each level was randomly assigned (with equal probability) a grade between 5 and 15 grams of gold per tonne of ore. This range of grades is realistic for a typical gold mine; and the fact that the values assigned are not real does not compromise the evaluation of our optimization model.

Table 4.1: Ore and waste values at each level of the case study at (RLGM).

Level	Gold Ore (tonnes/day)	Grade (g/tonne)	Gold (ounces/day)	Waste (tonnes/day)
16	174	7.1	43.7	120
17	291	5.6	57.4	126
18	208	10.0	73.4	142
19	318	11.7	130.9	155
20	258	10.6	96.3	134
21	291	14.1	145.3	133
22	277	11.3	110.5	145
24	168	6.5	38.9	112
26	259	8.9	81.6	119
27	215	11.6	87.6	201
28	251	9.9	87.5	152
29	264	13.7	127.3	187
31	226	8.5	67.5	111
32	171	9.0	54.6	86
33	205	9.7	70.2	129
34	245	13.2	114.3	116
35	185	5.6	36.4	179
36	179	6.3	40.1	83
37	260	5.4	49.4	184

The optimization model was built using MPL® software and solved using the branch and bound algorithm of CPLEX® 12.0 on a Windows 10 operating system using an Intel CORE i7 CPU with 16 gigabytes of RAM. The model had 133 binary decision variables and 226 continuous flow variables. All instances were solved in less than 30 seconds.

4.4 Results

The objective function values resulting from the application of the optimization and greedy heuristic models to the case study are presented in Table 4.2 (below).

Table 4.2: Results of the optimization and greedy heuristic models compared.

	Gold (ounces/day)	Ore (tonnes/day)	Waste (tonnes/day)	Total (tonnes/day)
Optimization Model	774.8	1,993	993	2,986
Greedy Heuristic 1	675.6	1,292	921	2,213
Greedy Heuristic 2	731.1	1,708	1,007	2,715

Table 4.2 yields several observations. First, the objective function (i.e., maximize gold ounces mined per day) of the optimization model is greater than the objective functions of both greedy heuristics: it is 14.7% higher than greedy heuristic 1 and 6.0% higher than greedy heuristic 2. These results indicate the benefit of formulating and using an optimization model for solving this problem versus using the current greedy heuristic. Second, Table 4.2 also shows that greedy heuristic 2 achieved a higher objective function than greedy heuristic 1 by exceeding its waste limit (of 1,000 tonnes/day) by 7 tonnes per day; and that, even after exceeding its waste limit, the objective function of greedy heuristic 2 was 6% lower than that of the optimization model. These results therefore show that the value of a solution to this problem depends not only on the optimal removal of gold ore alone; but also, on the optimal removal of ore and waste simultaneously. Third, Table 4.2 also shows that the solution of the optimization model came closest to maximizing the capacity of network's final node (i.e., 3,000 tonne/day capacity of the elevator carrying material to the surface). This can be seen by the values under the total tonnes removed per day where the optimization model's solution used 96.5% of the capacity, while greedy heuristic 1 used only 73.8% and greedy heuristic 2 used 90.3%. These differences in capacities used shows the

importance, in this problem, of packing the elevator to the surface as closely to its capacity as is possible by using an optimization model based on the knapsack model.

The solutions of the optimization model and greedy heuristics are presented in Table 4.3.

Table 4.3: The solutions of the optimization model and greedy heuristics.

Level	Optimization Model	Greedy Heuristic 1	Greedy Heuristic 2	Grade (g/ton)
16				7.1
17				5.6
18				10.0
19	x	x	x	11.7
20				10.6
21	x	x	x	14.1
22	x			11.3
24				6.5
26	x			8.9
27		x	x	11.6
28				9.9
29		x	x	13.7
31	x			8.5
32	x		x	9.0
33	x	x	x	9.7
34	x	x	x	13.2
35				5.6
36				6.3
37				5.4

Table 4.3 presents the locations of the levels selected for mining in each day. Note that the selected blocks at each level did not change over the days of scheduling because of their great magnitude (in tonnes) relative to magnitude (in tonnes) of material removed daily. Table 4.3 yields 3 observations worthy of note. First, one can observe the effect of the first flow constraint (between levels 29 and 31 as shown in Figure 4.3), of 1,800 tonnes of total material flow per day, on the three different solutions. Given the parameters on ore and waste material at each level (in

Table 4.1), one can see that the solution of the optimization model in Table.4.3 moved 1,606 tonnes of material and 491 ounces of gold from level 29 to 31; and the solution of the greedy heuristic moved 1,689 tonnes of material and 468 ounces of gold. Hence, the capacities between levels 29 and 31 was more closely met by the greedy heuristic than by the optimization model. This difference indicates that the optimal solution does not need to maximize the flow of material or gold through this capacity constraint (between levels 29 and 31) as it does through the final capacity constraint (after level 37). Hence, in this model of the problem, the final capacity constraint alone functions as a knapsack constraint, i.e., a constraint by which as much gold as possible must flow subject to a capacity limit on total flow of material (tonnes/day).

Second, Table 4.3 shows that the optimal solution contains levels of a lower grade of ore than the solutions of the greedy heuristics. The average grade of ore for the levels selected in the optimal is 10.8 g/tonne; while the average grade per level for the solutions of greedy 1 and greedy 2 were 12.3 and 11.9 g/tonne, respectively. The lower grade selected in the optimal solution was facilitated by more closely packing the total material capacity constraint of 3,000 tonnes/day, thus enabling the movement of more material and therefore more total gold per day than the greedy heuristics. Hence, the results in Table 4.3 illustrate how the optimal solution represents a more successful resolution of the trade-off implicit in this problem; viz., the problem of packing as much gold as possible into the terminal node of the network subject to capacity constraints in the transportation network.

Third, from Table 4.3 one can observe that the optimal solution contained 8 levels for operations while the solutions to the heuristic models required only 7 levels for operations. The extra level of operation contained in the optimal solution entails a higher operational cost and the decision-maker must evaluate whether the additional cost of operations is worth the additional flow of gold accompanying this solution. In this case, the trade-off between the optimal solution and the solutions of heuristic 1 and 2 implies that: an increase in gold moved to the surface, by 14.7%, requires an extra 2 levels of operation compared to heuristic 1; and an increase in gold mined, by 6.0%, requires an extra one level operation compared to heuristic 2 (which are also has excess waste mined).

4.5 Discussion

From the results, we find several points deserving discussion: first, the merit of the greedy heuristic used in this paper; second, the practical benefits of using this model; and finally, some thoughts on the benefits of developing an operational-scale optimization model for underground mining.

First, an evaluation of the greedy heuristic versus the optimization model is required. To do this, it should first be noted that a greedy heuristic has been used, for many decades, to solve multiple versions of the knapsack model (Pisinger 1999) and has produced useful results with quick computational times (Akcay et al. 2007). In other words, by using the greedy heuristic method to provide results with which to compare and evaluate our optimization model, we have not selected a weak and insignificant method. Greedy heuristics have been used, in practice, to solve large instances of the knapsack problem (Ferdosian 2016). The operational mine-level scheduling problem modeled in this paper is not likely ever to have a problem instance so large that it will require a greedy heuristic to solve it. This is because the binary decision variable, used in this model, represents a mine's level; and there would need to be in excess of 5,000 mine levels for a problem instance to be computationally infeasible for an optimization model which is NP-hard. Hence, it unlikely that the benefits of using an optimization model instead of a greedy heuristic, to solve the model in this paper, will ever become computationally infeasible.

Second, it should be observed that the results illustrate two practical benefits of using the optimization model instead of the greedy heuristic. The first benefit is the increased value of the objective function. The objective function of the optimization model was 14.7% and 6.0% higher than the objective function of greedy heuristics 1 and 2, respectively. The practical benefit of this is an increase in the mine's economic productivity. The second benefit is an improved scheduling of waste removal. The results produced by the greedy heuristics show the awkward predicament that arises when a greedy heuristic is used to schedule the removal of both ore and waste; i.e., either underutilization or overutilization of the mine's transportation capacity occurs, when compared to the solution for the optimization model. For example, Table 4.2 shows that heuristic 1 underutilized the mine's transportation capacity because it was forced to stop adding levels to be mined when the waste to be moved reached 921 tonnes (i.e. 92% capacity) and this meant that total material (i.e., ore + waste) to be moved reached only 2, 213 tonnes (i.e., 74% of capacity). Table 4.2 also shows that, for heuristic 2, the mine's capacity to remove waste was slightly exceeded. In practice, the excess waste is placed in temporary underground storage and when the capacity of this underground storage is exceeded, then the scheduled flow of ore through the system must be interrupted so that the excess waste can be removed. These intermittent interruptions of the movement of ore to the surface have the effect of intermittently under-utilizing the ore-processing facilities at the surface of the mine. Hence, an additional practical advantage of using the optimization model is not only that it maximizes the mine's capacity to move material, but that it does so without intermittent interruptions caused by the stockpiles of waste.

Finally, the results illustrate the benefits of developing an operational-scale model for an underground mine. The reason operational-scale optimization models have not been widely used in the underground mining industry is because operational problems in underground mines are less generic than tactical models. That is, different types of mines have constraints or objectives that are peculiar to that mine type, and an optimization model would therefore need to be tailor-made for that type of mine in order to plan for operations as a result, the development and use of operational-scale models has not been broadly used and heuristic approaches have relied upon to generate solutions (Chowdu et al. 2021). The results in this paper illustrate the scale of the economic benefits that can be gained by developing and using an operational scale model versus the heuristics.

4.6 Conclusion

It is an especially difficult task to choose the best places for operations inside an underground gold mine. It cannot be resolved by only selecting the levels with the highest grade of ore because the underground mine's ore transport network has a range of capacity limitations that may prohibit the immediate mining of all the levels with the highest grade. To address this problem, a new optimization model was developed and evaluated in this paper. The optimization model was applied to a gold mine in Red Lake, Ontario, Canada. Its results were compared to heuristic decision-making rules currently used in the case study. The results illustrate that an increase in daily productivity between 6% and 14.7 % resulted from using the optimization model versus the currently used heuristic method. These results indicate that the development of new optimization models for underground mining problems can be a field of study with important economic consequences.

Chapter 5

Contributions and Future Work

5.1 Contributions:

This work provides modeling advance innovation in the operations research methods for solving major outstanding problems in the gold mining industry and highlights the main motivations and impacts of solving these problems in order to improve strategic planning in that critical industry. This dissertation applies certain aspects not covered in previous studies to the case study. Thus, this research will contribute to treating three major problems, each outlined in a separate chapter in this thesis, as well as the application of these models in the case study at Red Lake Gold Mines owned by Newmont Goldcorp Inc. It is our hope that, in collaboration with our industrial partner, the work of this thesis will not only be of academic merit but will also have practical value to the industrial partners, as well as to the gold mining industry in general. Therefore, our new contribution in this research will be as follows:

Problem 1: Solving the problem of fresh-water usage during the gold processing stages:

- Addressing environment sustainability constraints.
- Using an optimization model, which had never been applied in a gold mine, as the case study.
- Reducing fresh-water consumption costs.

Problem 2: Solving the truck and shovel dispatch scheduling problem, to maximize quantities value for gold that is transferred > optimal transportation costs > shovels used > trucks used and determine their direction (what level the trucks should proceed to) as goal variables integer:

- Linking short-term scheduling with the trucking operations with the quantities of gold transported, precedence transporting, and operating costs in specific ore extraction levels (between levels 39 and52), at a depth of up to 2.4 km.

- Designing and applying a special optimization model using a mathematical integer programming model, depending on real data for gold production in the first quarters of 2019, at an underground gold mine as the case study, which has never been applied.

Problem 3: Solving the short-term production scheduling problem of ore-waste material flow used in underground gold mines operations:

- Linking the short-term scheduling of ore material flow operations, between different levels of ore extract with the longer-term tactical plan.

- Designing a special model for the problem, given that there exists no commercial optimization software that is universally employable in all underground mines for optimizing production scheduling.

5.2 Future Work:

The gold mining industry in Canada was, and is still, an important component of the country's national economy. Remaining competitive in a global marketplace requires continual development into the innovation of optimization models; moreover, improvements in software (i.e., faster and better technology). There are many complex and dangerous activities and operational-economic constraints which have caused problems in its manufacturing and operational systems, as well as a scarcity of optimization models applied in the gold mines and gold processing stages, as we indicated in the literature reviews.

The models applied in the chapters have provided the goals of this thesis by solving four different and important problems in Newmont Goldcorp Inc. at Red Lake Gold Mines (RLGM) in Northwestern Ontario, as the case study, by using O.R. methods. However, there are still several issues related to them, which may be subject to future research, as follows:

- Emphasis on the use of O.R. methods and applying them in order to optimize gold mines operations and their processing stages. Due to the scarcity of similar data, we chose the Red Lake Gold Mines (RLGM), owned by Newmont Goldcorp Ltd. as a realistic case study.
- An increased emphasis on sustainability and finding the best ways to reuse spent water in gold processing operations.
- Focus on the optimization of the short- and long-term planning models, especially in underground operational processes, to help the future vision, and to improve optimizing productivity and achieve sustainability.
- The crushing stage at Red Lake Gold Mine has suffered an operating power consumption issue during the liberation of ore gold material from waste, which is required for gold processing operations. We attempted to solve this problem in this study, but given the absence of data, our limited understanding of the issue, and the repeated lack of response to the visit request. Thus, future studies can follow up research on the operational time of power consumption problem used in the crushing stage during gold ore processing

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