THE EFFECT OF SIZE, VALUE, AND ARRANGEMENT ON PERCEIVED NUMEROSITY

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A THESIS
SUBMITTED TO THE FACULTY OF ARTS
IN PARTIAL FULFILLMENT OF THE
REQUIREMENTS FOR THE DEGREE OF
MASTER OF ARTS

DEPARTMENT OF PSYCHOLOGY

LAKEHEAD UNIVERSITY

THUNDER BAY, ONTARIO

AUGUST 1972

Abstract

Numerosity (perceived quantity) is shown in four experiments to vary with several factors: item-size, item-value, pattern size, and possibly the arrangement of items within patterns.

In Experiment One, evenly arranged circular patterns of large and of small discs were compared. More subjects attributed greater numerosity to the small discs when both patterns were equal in area. But when the large disc pattern was enlarged relative to the small disc pattern, more subjects attributed greater numerosity to the large discs. The significant results ($p<0.025$) for the item-size/pattern area effect were interpreted as follows: numerosity varies directly with pattern area and inversely with item-size. The perceptual numerosity of a given stimulus array results from the algebraic summation of both effects.

In Experiment Two, a significant numerosity illusion was also demonstrated for the item-value factor. Half-dollars were judged to be significantly more numerous ($p<0.01$) than an objectively equal quantity of pennies. But since pennies and half-dollars vary in size as well as in value, the illusion may be due at least in part to item-size/pattern area considerations. To eliminate the

effect of these extraneous variables, coins of equal size but different value must be compared. Canadian pennies and dimes are approximately equal in size. In Experiment Three, these coins were compared for numerosity and dimes were judged to be significantly more numerous ($p<0.001$). Since higher value items were overestimated both when they were larger and when they were smaller than the lower value items, it was concluded that the value factor accounts for a significant numerosity illusion independent of the item-size effect.

There is some evidence that numerosity is also dependent upon the randomness-regularity dimension of the spatial arrangement of items within patterns. In Experiment Four, a random arrangement of items appeared significantly less numerous ($p<0.01$) than an objectively equal quantity of similar items arranged evenly throughout concentric rings. But in this experiment the random stimulus was smaller in overall pattern size and the illusion may be the result of the pattern size effect.

Acknowledgement

I would like to thank Professor Norman Ginsburg whose experience and direction has made this paper possible. I would also like to thank Dr. Frank Colman for his advice in the preparation of the manuscript.

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Introduction

Review of the Literature

Binet (1890) reported that a single juvenile subject, his four year old daughter, judged an array of larger stimuli to appear more numerous than an array of smaller ones. Using coloured discs as jetons or "counters", his generic term for stimulus elements appropriate for numerosity comparisons, he showed that 18 small counters, 25 mm in diameter, were perceptually equal to nine and a half 40 mm counters.

Binet interpreted these results as indicative of a developmental illusion--that they "...prove(d) only that the child when perceiving a large number of discrete objects has a very great difficulty in perceiving them other than as a solid entity" (Pollack and Brenner, 1969, p 88). Presumably Binet felt the illusion would disappear after the child developed a capacity for the discrimination of discrete numerosity in multi-element stimuli, but he did report that the same child could consistently discriminate between 18 and 17 identical counters.

Egon Brunswik (1956) described a similar item-size--numerosity illusion with adult subjects. Brunswik was primarily interested in describing the relationship of monetary value to perceived numerosity.

His design involved the comparison of coins of different values. But as with most currencies in common use, the coins of higher value were also of greater size, and Brunswik was obliged to conduct a control study in which plain discs that differed in size in a similar ratio as the coins were compared for numerosity.

For this purpose, Brunswik referred to the work of Zuk-Kardos, one of his students, whose research was described in Brunswik (1934). Using the method of constant stimuli, a standard pattern containing 40 smaller discs was compared for numerosity with varying quantities of larger discs, and Zuk-Kardos reported a size--numerosity illusion in which 34 of the larger discs were perceptually equal in numerosity to the 40 smaller elements of the standard. Thus the illusion described by Brunswik and Zuk-Kardos was qualitatively similar to the effect Binet described with the child. The Brunswik/Zuk-Kardos results suggest the general validity of the Binet (1890) illusion: that adults as well as children perceive large items as more numerous than smaller items.

Regarding the value--numerosity effect, Brunswik (1956) reported data by Fazil in which 40 low value, small coins were perceptually equal to 29 higher value, larger coins. Greater value, Brunswik believed, affected numerosity in the same way as greater size.

But conflicting evidence on the size--numerosity illusion is provided by Liebenberg (1914), by Mokre (1928), by Kasting (1935) and by the data of a recent experiment (Courtis, 1970; see also Courtis and Ginsburg, 1971).

In the Mokre (1928) research, circular patterns of different size discs were compared for numerosity and the smaller items were reported to be perceptually more numerous. Kasting (1935) agreed with Mokre (1928) that adults underestimate large items, and showed a developmental change in the qualitative nature of the illusion. Kasting replicated the Binet illusion with children but reported the item-size--numerosity illusion in adults to be qualitatively opposite to the effect described by Brunswik (1934).

The results of Liebenberg (1914) add further complication to the problem. Liebenberg found that large dots were underestimated by children as well as by adults.

In the Courtis (1970) research, the Brunswik/Zuk-Kardos experiment was replicated with a rigorous, modern psychophysical technique and a diametrically opposite interpretation of the qualitative nature of the size--numerosity illusion was drawn. Courtis (1970) specified the illusion as follows:

> When two equivalent quantities of different sized

dots are presented for comparison, that pattern composed of the smaller dots will be judged the more numerous.

The Courtis (1970) technique involved the comparison of disc patterns composed of elements differing in area as a ratio of 1:4, the same ratio of element size differential used in the Zuk-Kardos experiment. It was found that the tendency to overestimate small-dot-numerosity as compared with large-dot-numerosity was highly significant ($p<0.001$).

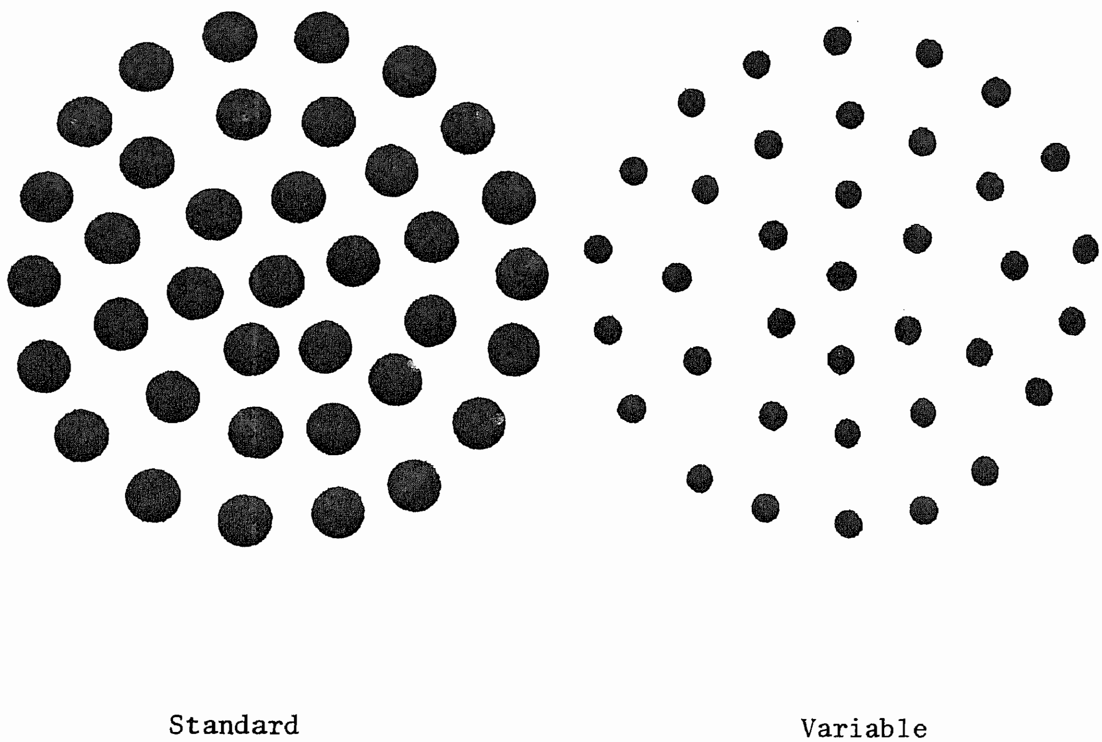
But procedural differences between the techniques of Brunswik and Zuk-Kardos, and of the Courtis (1970) experiment may have been reponsible for the qualitatively opposite illusion values.

First, in the Zuk-Kardos experiment, the standard was composed of the smaller sized elements; in the 1970 experiment, the standard was composed of the larger elements. Mokre (1928) used both conditions.

Secondly, in the 1970 experiment the stimulus patterns consisted of even, concentric arrangements of the disc elements; see Figure 1. This systematic patterning of the items is in contrast to the arrangements used by Brunswik and Zuk-Kardos and also to those of Binet (1890). Brunswik (1934) did not give an

Figure 1

The Stimuli of the Courtis (1970) Experiment



account of a systematic method used to generate the arrangements, and one is lead to believe that none existed. The examples of the Zuk-Kardos stimuli presented in Brunswik (1934) are reproduced in Figure 2. If we assume these examples to be representative of Zuk-Kardos' stimuli, then it appears reasonable to conclude that she used arrays in which an attempt was made to preserve an overall homogeneous distribution, or even-density patterning of the stimulus elements. No other indications of systematic imposition are obvious in the Brunswik/Zuk-Kardos research.

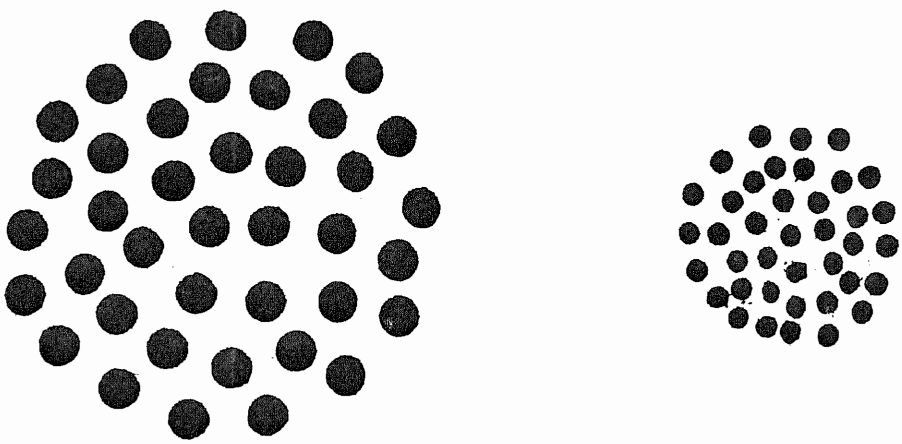
Mokre (1928) did not include an illustration of his stimuli but from his description, they must have resembled the Brunswik/Zuk-Kardos patterns. According to Mokre, his stimuli were prepared such that the discs were equidistant, arranged as indifferently as possible in an approximately circular form.

Thirdly, a major point of difference between the stimuli of Brunswik/Zuk-Kardos and those of Courtis (1970) lies in the fact that in the stimuli of the latter experiment the distance between borders of adjacent elements was greater in the small item patterns than in the large item patterns.

In the 1970 experiment, both standard and variables were of equivalent overall dimensions. Since the distance between borders

Figure 2

The Stimuli of the Zuk-Kardos Experiment as Represented in Brunswik (1934)



Standard Variable

of adjacent items varies directly with overall pattern area, and inversely with item-size when pattern area is held constant, the larger discs of the standard were more densely packed together than the smaller discs of the variables; that is, divergent element density (the ratio of filled to unfilled space) is a necessary consequence of varying item-size in equal area patterns of different sized discs.

The Mokre (1928) research used stimuli in which inter-item distance was systematically varied. But for his estimate of the perceptual effect of item-size on numerosity, Mokre used stimuli which resembled those of the 1970 experiment. That is, in the Mokre stimuli, the distance between centres of adjacent elements was the same in both the large disc, and small disc patterns.

Brunswik hinted in a footnote to his 1934 text that divergent element density between patterns of different sized discs would lead to a numerosity illusion independent of the item-size effect. In Brunswik's method, the large item patterns were analogous to photographic blow-ups of the smaller disc patterns; that is, the proportion of filled to unfilled space (element density) was constant between standard and variables.

To quote Brunswik:

> Very important in these experiments is the fact that the relative compactness of distribution of the elements is the same for standard and variable. Neglect this and you have admitted the source of a new illusion and obtain different results--as a few experiments carried out in this way distinctly showed. (Brunswik, 1934, p 142)

Brunswik (1934) did not refer to Mokre's data published in Germany six years before, but his "preliminary experiments" may have corroborated the Mokre (1928) description of the item-size effect (or he may have found no illusion at all).

A fourth point of difference between the Brunswik/Zuk-Kardos experiment and the Courtis (1970) investigation may lie in methods of presentation of the stimuli. In the modern research, the stimuli were systematically presented in the same manner for every subject using a slide projector and an electric timer to control exposure and inter-exposure durations. Mokre (1928) also used projection.

The procedure of the Brunswik/Zuk-Kardos experiment was not clearly specified, but it seems likely that some means of manually setting out stimulus aggregates was employed. It is believed that the subject's task involved the simultaneous comparison of the stimulus patterns and that stimulus exposure was brief enough to preclude explicit counting.

In Mokre (1928) and in Courtis (1970) successive comparison of stimulus patterns was used. Mokre used 0.75 second exposures to each stimulus; Courtis used two second exposures.

In passing, it should be noted that Binet's jetons were, in his own words, merely "...placed before the child." The possibility of extraneous variability contaminating his results is indeed credible and incompatible with Binet's otherwise elegant science. In the 1970 experiment, such considerations were taken under control.

A fifth major point of difference between the 1970 experiment and the earlier investigations may lie in the fact that the early studies relied upon inadequately small samples. Brunswik did not specify sample size and Binet, rather questionably, relied upon a single subject for his conclusion as to the qualitative nature of the size-numerosity illusion. It is perhaps significant that in the 1970 study, a full 26% of the 64 subjects responded to the stimuli in the same manner as Binet's daughter and Zuk-Kardos' sample of unspecified size.

Another possible point of difference between the Brunswik/Zuk-Kardos investigation and the 1970 experiment may lie in the nature of the stimulus patterns. Courtis (1970) used white items on a black background. It is believed that Brunswik/Zuk-Kardos used black items on a white background.

Present Investigation

The intent of this thesis was to determine conclusively the effect of density, size, value and arrangement on perceived numerosity.

Since Brunswik (1934) and Courtis (1970) both indicated that the divergent element density factor may be associated with a substantial and previously undefined effect on perceived numerosity that factor is brought to test in two of the four experiments of this thesis.

Element density in concentric ring patterns is of two types. Intra-ring density is the simple figure-ground relationship of filled to unfilled space within a given ring. Intra-ring density varies directly with increasing intra-ring numerosity; thus, divergent intra-ring density is a necessary consequence of varying numerosity in a ring of constant diameter.

Inter-ring density refers to the ratio of item diameter to the minimum possible distance between stimulus items in adjacent concentric rings. It is this second dimension of pattern density that is assumed to be of greater psychophysical interest since it can be maintained constant while intra-ring numerosity is systematically varied. As well, intra-ring density is correspondingly increased as inter-ring density is increased, since circumference varies directly with diameter.

Equivalent inter-ring density between patterns of different size elements can only be maintained at the expense of overall pattern area equivalence. In these experiments, a systematic compromise between pattern area equivalence and element density equivalence was used. Standards of two degrees of density differential relative to the variables were used, A and B.

In the Series A standards, the distance between centres of adjacent items was equal to the distance between centres of the adjacent smaller items in the variable of equal numerosity.

In the Series B standards, the distance between borders of adjacent items was equal to that of the variable of equal numerosity.

The Series A standards were approximately equal in overall pattern area to the variables.[1] The Series A patterns could be superimposed over the variable of equal numerosity, whereas the Series B standards always formed a total pattern of considerably greater area. Element density (the ratio of filled to unfilled space was always systematically greater in the A standards.
A comparison of illusion values found with both types of standards

[1] see Appendix 2

was used to determine the existence of a significant systematic effect of differential density on perceived numerosity.

The experimental determination of the quantitative relationship of size to perceived number was a central topic of this thesis, and the second variable of interest in Experiment One. The development of the function of this quantitative relationship was attempted by the controlled manipulation of the size factor. That is, the element size of the variable stimuli was held constant while the element size of the standards was systematically varied. Thus, standards composed of discs varying in area relative to the discs of the variables as ratios of 1:2, 1:4, and 1:9 were used with different subjects. A systematic trend for increasing size divergence to produce some corresponding, systematic effect on illusion value was the object of the analysis of the data collected in Experiment One.

The effect of item-value on perceived numerosity was a third factor of interest to this thesis. Brunswik (1956) described the effect of monetary value on perceived numerosity. Using patterns composed of Turkish coins, "quarters" and "half-nickels" (two and a half cent pieces) he reported a high illusion value of 29/40; that is, 29 of the higher value coins were perceptually equal to 40 of the lower value coins.

In the second experiment of this research, the Brunswik coin experiment was replicated with Canadian coins--half-dollars and pennies--using the Courtis (1970) technique. Experiment One included a size divergence comparison of 2:1 by area--the same ratio which distinguishes half-dollars from pennies. Thus the item-value--numerosity effect was isolated by the analysis of the results of the coin experiment together with the results obtained when plain discs which vary in size in the same ratio as the coins were compared for numerosity.

In order to further isolate value from size, coins of equal size but different value must be compared. Canadian pennies and dimes are approximately equal in size (actually dimes are about 10% smaller). Therefore, in Experiment Three, actual patterns of dimes and pennies were compared for numerosity. In this experiment, because of the similarity of the coins and because of the inadequacy of simple photographic techniques to reveal the difference, actual physical objects were used.

A fourth topic of this thesis concerns the perceptual effect of element arrangement on numerosity. In the present experiments, the method of generating the systematic concentric ring arrangements of stimulus items is specified in some detail. By contrast, the patterns of Binet, Mokre and Brunswik were not described in sufficient detail to permit of their exact replication. It is apparent, however, that the earlier experimenters attempted to generate arrangements in which the distribution of stimulus items was as homogenous as possible.

In Experiment Four, the 11 concentric ring variables of the first experiment were compared with a "random standard" composed of equal sized discs arranged without regard to homogeneity of distribution.

In summary, four experiments embody the empirical substance of this thesis. Experiment One investigated the quantitative relationship of item-size to perceived number. Experiment Two investigated the value-numerosity relationship. Both included control conditions to determine the effect of density differential between standard and variables. Experiment Three investigated the isolated effect of value on perceived numerosity. Experiment Four investigated the effect of element arrangement on perceived numerosity where a random pattern was compared with concentric ring patterns of similar overall size.

Method and Results

Experiment One: Large Discs versus Small Discs

Subjects:

One hundred and two students from the intramural, introductory psychology course served as subjects. These young adults, mostly freshmen, volunteered for participation and received a one percent bonus on their final course grade for serving in the experiment.

Apparatus:

The apparatus used in the experiments included the following equipment. A Kodak Carousel 800, 35 mm slide projector was used with four Carousel slide trays, and a Buhl Superwide, 2.0 inch focal length, wide angle lens to facilitate full screen reproduction of the slides in relatively small laboratory areas where projection distances were limited. A Hunter, Model 111-C interval timer was used to advance the projector at a constant, reliable rate.

The timer was connected to the projector by means of a modification to the external, manual changing control of the projector. This permitted direction reversal of the slide changing mechanism.

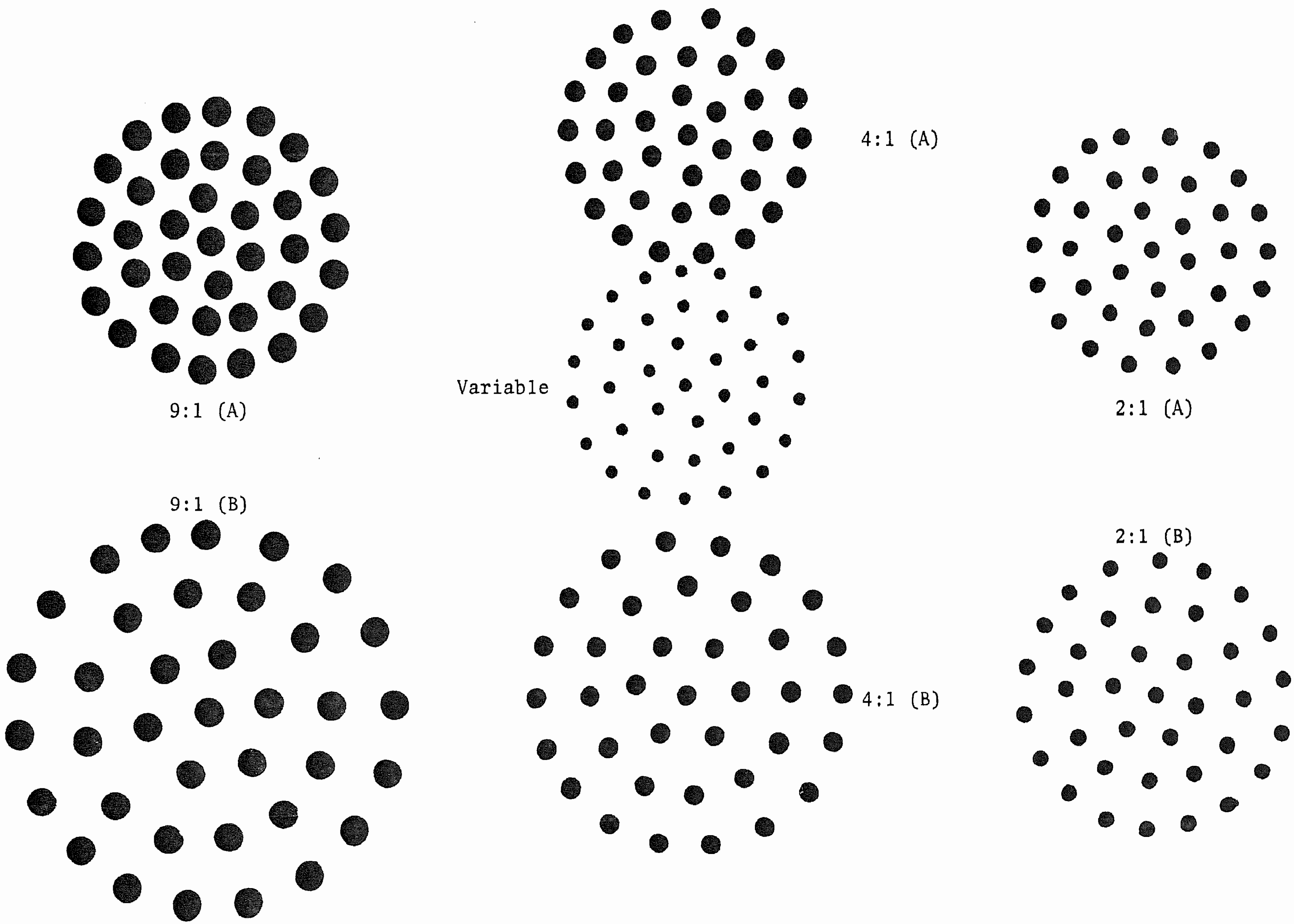
Stimuli:

The stimuli consisted of 35 mm slides which were prepared as follows. Canadian pennies, 19.05 mm in diameter, were coated in dull black latex paint and photographed on a white Bristol board background. A Nikon F (f 1.4, 50 mm lens) 35 mm SLR camera was used with Kodak Panatomic X (ASA 32) negative film. The dull black coating virtually obscured the identity of the resultant white discs of the developed negatives. The resultant background was an evenly dark field of constant brightness. Brightness measurements made with a Macbeth illuminometer showed the luminance of the discs to be 0.59 ft-L, and the background to be 0.1 ft-L. The negatives were mounted in glass slide jackets.

Six standard stimuli were prepared and one set of 11 variables. These patterns, shown in Figure 3, consisted of quantities of discs arranged evenly throughout concentric rings. The standards all contained 37 discs distributed evenly throughout three concentric rings. The outer, intermediate and inner rings contained 18, 12, and six discs respectively; the 37th disc was placed at the pattern centre. Since the radii of these rings varied as the ratio 3:2:1 the circumference of the rings varied as the same ratio. Therefore

Figure 3

The Six Standard Stimuli and the Variable
of Equal Numerosity, Experiment One



intra-ring element density was constant within each standard.

The 11 variables contained 28, 30, 32, 34, 36, 37, 38, 40, 42, 44, and 46 discs respectively. These patterns were prepared such that whenever two discs were added to the outer ring, the next was added to the intermediate ring. Appendix 1 lists the 11 variables and specifies the number of discs in each of the outer rings.

The inner ring of the variables always contained six discs, in deference to the probability that the central seven items are perceived by a different perceptual process than are the discs of the outer rings; that is, subitization as described by Kaufman, Lord, Reese and Volkmann (1949). Any alteration of this central configuration would probably result in an obvious change in perceived numerosity.

Elements of the standard patterns were always larger than the elements of the variables. Three size difference ratios were used--2:1, 4:1, and 9:1 by area. This was accomplished by photographing the patterns from different heights. The variables were photographed from a height of 1950 mm. The standards were photographed from heights of 1380, 975, and 650 mm.

The six standard patterns were of two types: Series A, and Series B. Each ratio of element size differential was represented once in each series.

In the Series A standards, the distance between centres of adjacent items was equal to that of the variable with 37 items. In the Series B standards, the distance between borders of adjacent items was equal to that of the variable with 37 items. That is, the A Standards could be superimposed over the variable of equal numerosity, whereas the B Standards always formed a total pattern of considerably larger size. Therefore, element density (the ratio of filled to unfilled space) within the B Standards was always less than the element density of the corresponding A Standards. Appendix 2 shows the dimensions and relative element densities of the six standard stimuli and the variable of equal numerosity.

The subject and experimenter sat at a table on either side of the slide projector and the stimuli were viewed from a screen located approximately two meters away. The projected diameter of the outer ring of the variable pattern was 430 mm. The visual angle subtended by this diameter is approximately 11°. The relative size and approximate visual angles subtended by the outer rings of all patterns and by the items themselves is also reported in Appendix 2.

Procedure:

By the method of constant stimuli, one of the standards was paired with each of the 11 variables and the subject was instructed (see Appendix 3) to make relative numerosity judgments for each pair of stimuli. Each of the six standards was used until 15 subjects reported acceptable PSEs (see Treatment of the Data).

Standard-variable pairs were presented successively. The subject responded by writing "one" or "two" on a score sheet according to which stimulus appeared more numerous. Each subject was shown four different series of the 11 standard-variable pairs.

In order to acquaint the subject with the range of the stimuli the first four pairs were considered "warm-ups" and were not included in the transcription of the subject's response protocol for Series One. The subject was not told that these four initial pairs were not formally part of the experiment, nor were these first pairs distinguishable from the next 11 test pairs of Series One. Each of the warm-ups demanded only gross discrimination; that is, each involved the comparison of obviously different quantities. Preliminary investigation has shown that this initial overall acquaintance with the stimuli can avoid the formation of a predisposing response set--such as a tendency to respond either always to the standard or always to the variables.

The order of the variables over the 11 pair positions in each series was arbitrarily composed so that no critical comparisons (those expected to be near the PSE) would occupy positions at the first of last of the series where random errors would seem most likely to occur. A different arrangement of the variables was used in each of the four series.

The orders of standard-variable, within pairs, were selected as follows. In the four series, each variable was presented a total of four times--once in each series. Twice it preceded the standard, and twice it followed the standard. In this way, the sequential position factor was brought under control. Appendix 4 shows the arrangement of the stimuli in all four series.

The two slides of each pair were placed in adjacent slots of the circular Carousel slide trays; one slot was left empty between pairs. The timer was set for a three second interval. Since the change mechanism of the projector requires approximately one second for a complete cycle, the subject was given two-second exposures to each stimulus, with one-second intra-pair intervals. The inter-pair interval of four seconds consisted of two changes of the projector plus a two-second blank exposure. During this latter interval, sufficient light was reflected from the screen to enable the subject to record his response to the previous pair.

After completion of the initial 15 pairs (Series One), the subject was given a new score sheet and his completed sheet was placed out of sight. The subject was then read an abbreviation of the instructions and Series Two commenced. With the completion of Series Two, the subject was given a 10 minute rest period outside the laboratory.

Following this rest period the subject was re-read the abbreviated instructions and was given Series Three and Series Four. A short rest period of approximately two minutes duration was allowed the subject after Series Three.

Treatment of the Data:

The raw data were re-ordered and transposed into meaningful form as follows.

First, a subject's score sheet was interpreted in terms of the numerosity judgments he had made. An example of a raw score (Series One) along with its interpretation follows.

Variable	30	42	37	34	38	40	28	36	46	44	32
Temporal position of variable within pair	2	1	2	1	2	2	2	1	1	2	1
Response of Subject	1	1	2	1	2	2	1	1	1	2	1
Relative perceived numerosity of variable	-	+	+	+	+	+	-	+	+	+	+

Note that "-" indicates that a given variable was reported to contain fewer discs than the standard; similarly, "+" indicates that a variable was perceived as containing more discs than the standard.

The final step in the interpretation of a subject's response protocol involved the rearrangement of the variables in numerical order, as follows.

Variable	28	30	32	34	36	37	38	40	42	44	46
Judgment	-	-	+	+	+	+	+	+	+	+	+

In this example, the subject's PSE would be taken as 31--the midpoint of the interval over which he reported a change of perceived, relative numerosity of the variables.

A mean value for the illusion with these stimuli was taken as the mean of all 15, first acceptable PSEs reported for each standard.

To be acceptable, results were required to meet the criterion of internal consistency, to be rational and to form a Guttman Scale. Rationality refers to that trend in a response protocol where the stimuli of greater physical quantity were identified as being of greater numerosity than the stimuli of lower physical quantity--a concept incorporated in the broader criterion of internal consistency.

A description of the criterion of acceptable internal consistency follows. Perfect internal consistency implies that a given protocol contains no more than two sets of responses: a set of "-" responses, one member of which was made to the variable 28, and a set of "+" responses, one member of which was made to the variable 46. An example of a perfectly internally consistent protocol follows.

Variable	28	30	32	34	36	37	38	40	42	44	46
Judgment	-	-	-	+	+	+	+	+	+	+	+

But to be acceptable, a given protocol was not required to be perfectly internally consistent; that is, an acceptable protocol could include a single Guttman Error (judgment reversal).

Thus, an acceptable protocol containing one Guttman Error may contain four sets of responses: two "-" sets, and two "+" sets. There are two prime sets in such a protocol: the "+" prime set includes the variable 46 and "-" prime set includes the variable 28. There are two secondary sets, one of which includes but a single member.

Two examples of acceptable protocols containing single Guttman Errors follow.

Variable	28	30	32	34	36	37	38	40	42	44	46
Judgment											
(Example A)	-	-	+	+	+	+	+	+	-	+	+
(Example B)	-	-	+	-	-	-	-	-	+	+	+

The PSE reported for protocols containing single Guttman Errors was based on the total number of "+" judgments. In Example A, PSE=33; in Example B, PSE=39.

Following the precedent of Segall, Campbell, and Herskovits (1963) it was decided that subjects may be legitimately expelled from the analysis of the data if their results still contain more than one Guttman Error after the test session of four runs through the stimuli. In the analysis of the four protocols of a single subject, the first protocol to meet criterion was accepted as the best indication of the perceptual effect of these stimuli. When a subject failed to respond with acceptable consistency by the fourth series, he was eliminated from the analysis of the data. Each standard was used until 15 subjects produced acceptable protocols.

The PSE for each standard with the variables was taken as the averaged value of the 15 first acceptable observations.

Chi square analysis was carried out on the observed PSEs for each standard type. The number of PSEs below objectivity were compared with the number of PSEs reported above the numerosity level of the standard. Yates' correction for continuity was not applied in this analysis despite the discontinuous nature of the data, since the expected frequencies of the resultant two-by-two contingency table, 22.5 per cell, far exceed the number specified by most writers as requiring the adjustment. According to Fergusson (1966), the correction is only required where any of the expected frequencies is less than five, and some writers suggest 10.

An analysis of variance was performed using the data collected with all six standards. The format of this analysis is shown below:

a n o v a

Source	d.f.
Ratios	2
Densities	1
R x D	2
Error	84
Total	89

This analysis was expected to detect if the two standard types (Densities) are the source of a significant perceptual effect. Also the analysis should detect any systematic tendencies in the quantitative effect of item-size (Ratios) on perceived numerosity.

Results:

Appendix 10 lists the PSEs of the 15 first acceptable response protocols for each of the six standards of Experiment One. Twelve subjects failed to reach the criterion of acceptability on at least one on the four series and were eliminated from the analysis. The breakdown by standard of these eliminated observations is also reported in Appendix 11.

Table 1 below shows the results of the analysis of variance of these data, where the effects of Ratios and Densities were tested for significance. Neither size divergence (Ratios) nor the differential density (Densities) factors were found to be significant in this analysis.

Table 2 reports average PSE for each of the six standards. This table also reports the distributions of PSEs by standard, and by standard type, according to the position of the 15 scores relative to the numerosity level of the standard. Combining these data according to standard type yielded the following results. For the three A Series standards, 16 subjects reported PSEs above objectivity, while 29 reported PSEs below the numerosity level of

Table 1

Analysis of Variance:
Experiment One

Source	df	MS	F
Ratios	2	37.15	0.396
Densities	1	147.30	1.595
R X D	2	148.05	1.583

Table 2

Distribution of Observed PSEs, Experiment One

Standard Ratio[1]	Standard Type[2]	Average PSE	Number of PSEs above 37	Number of PSEs below 37
2:1	A	35.1	3	12
2:1	B	37.3	10	5
4:1	A	37.0	7	8
4:1	B	36.5	6	9
9:1	A	36.5	6	9
9:1	B	37.1	11	4
All Standards		36.5	43	47
Combined A		36.2	16	29
Combined B		36.9	27	18

[1]Ratio of item-area of standard relative to that of variables

[2]Type A: pattern area of standard equal to variable
Type B: pattern area of standard enlarged relative to variable

the standard. For the three Series B standards, 27 subjects reported PSEs above the numerosity level of the standard, while 18 reported PSEs below objectivity.

Analysis of these two distributions as a two-by-two contingency table (see Table 3) yielded a significant value of Chi square ($\chi^2 = 5.38$; $p<0.025$). That is, perceived relative numerosity between patterns of different size items is significantly different for the two degrees of density differential.

Cursory examination of the results of this analysis suggests that the density factor does in fact exert a significant effect on the perception of numerosity even though this significance was not detected in the analysis of variance. This peculiar finding that Chi square should reveal significance in two distributions of scores when analysis of variance failed to do so was investigated further.

Figure 5 graphs frequency distributions of PSEs by standard type. It is evident that the distribution of PSEs of the Series B standards is bimodal. The presence of the lower mode does not affect the nonparametric Chi square analysis but renders the F test inapplicable, through violation of the assumption of normality of distribution.

Table 3

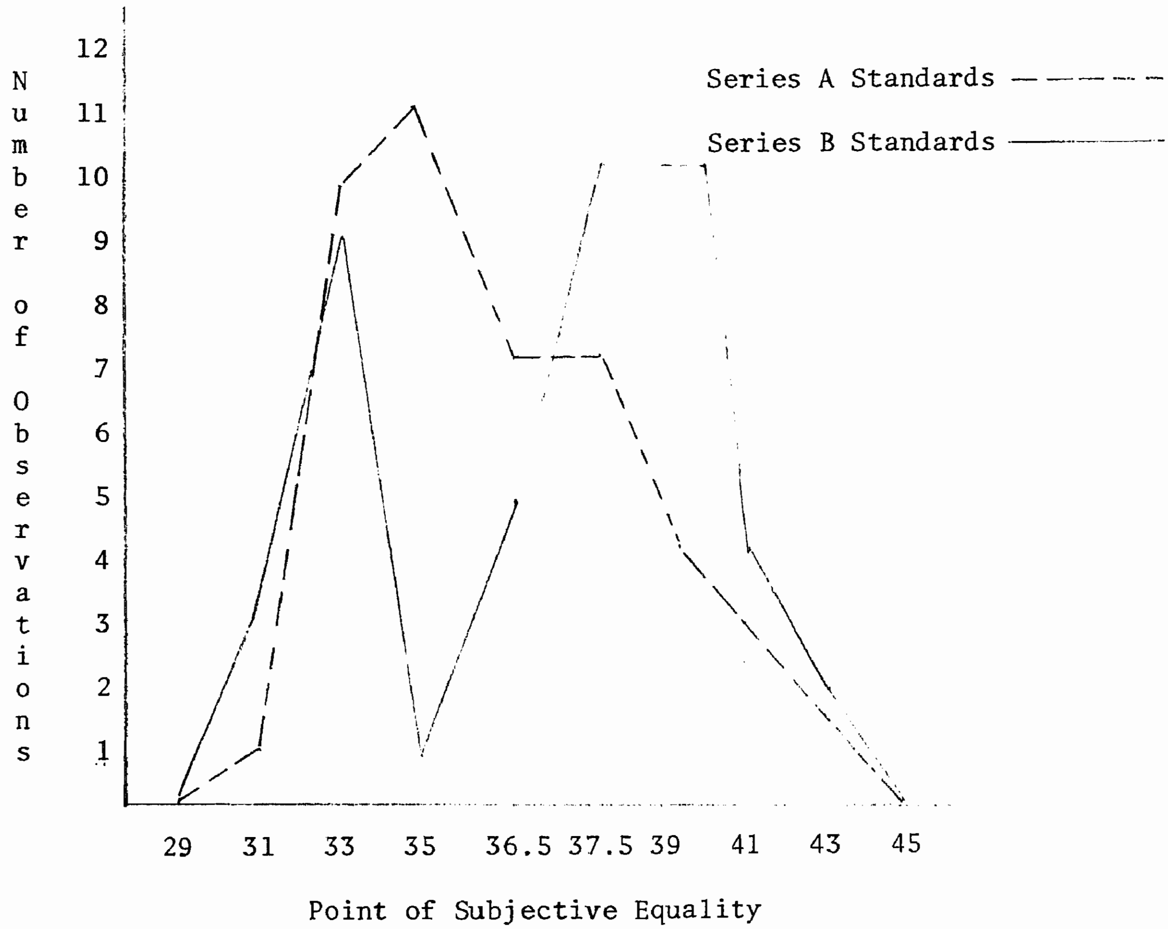
Chi Square Analysis of Distributions of PSEs
Experiment One

	Number of PSEs above Numerosity Level of Standard	Number of PSEs below Numerosity Level of Standard
Series A Standards	16	29
Series B Standards	27	18

$\chi^2 = 5.38$ ($p<0.025$)

Figure 4

Frequency Distributions of PSEs by Standard Type,
Experiment One



According to Edwards (1968; p 121) the F test is relatively insensitive to violations of the assumption of normality of distribution. Nevertheless, Chi square is clearly superior to analysis of variance in this application because no violation of assumptions underlying its use is involved.

Consequently the failure of analysis of variance to ascribe significance to the density factor may constitute a Type II error and may not substantially detract from the conclusion of the significance of the density effect, as determined by the Chi square test.

Experiment Two: Half-Dollars versus Pennies

Subjects:

Thirty-four students from the subject pool described for Experiment One served in this experiment.

Stimuli:

The stimuli consisted of patterns of Canadian pennies and half-dollars photographed from the same height. Since these coins vary in size as the ratio 2:1 by area (diameter of penny = 19.05mm; diameter of half-dollar = 27.01 mm), the monetary stimuli were made in the same dimensions as the disc stimuli of Experiment One which varied in size by this ratio. The coins were photographed on a black background with Kodak Kodachrome II (ASA 25) colour slide film.

Thirty-seven half-dollars composed the standard patterns. Two standards were used. In Standard A, the distance between centres of adjacent half-dollars was equal to the distance between the centres of adjacent pennies in the variable of equal numerosity. In Standard B, the distance between the borders of adjacent half-dollars was equal to the distance between the borders of adjacent pennies in the variable of equal numerosity. As with the stimuli of Experiment One, the A Standard could be superimposed over the variable of equal numerosity, while the B Standard formed a total pattern of larger overall size. Consequently, two degrees of element density were associated with the two standard stimuli.

The variable patterns composed of pennies ranged in numerosity from 28 to 54 coins. These were identical to the variables of Experiment One except that the discs appeared as new copper coins. The range of numerosity was extended from 46 to 54 in order to accomodate the effect of the illusion, since preliminary experience showed that the PSE for numerosity equivalence with these stimuli was commonly above 46. As well, the illusion value reported in Brunswik (1956) would suggest that the PSE for these stimuli would be above 46. Appendix 5 gives the composition by ring of each of the variables.

The projected diameters of the pennies and half-dollars were 32 and 45 mm respectively. Luminance measurements made with a Macbeth illuminometer showed the pennies to be 9.2 ft-L and the half-dollars to be 8.4 ft-L in brightness. The background luminance for the patterns was approximately 1.0 ft-L.

Procedure:

The procedure was basically similar to that of Experiment One. The subject was read a standard set of instructions (Appendix 6) describing the task. Subjects were required to identify the perceptually more numerous stimulus in a pair composed of a standard and one of the variables. The half-dollars were identified to the subject as quarters. Fifty cent coins are relatively rare in Canadian currency and it was arbitrarily decided that the subject's task would be simplified by identifying the stimulus items as the more familiar coins. No subjects suspected the change which is not surprising since the "heads" side of all Canadian coins are proportionately identical and differ only in diameter.

Two series of the 15 standard-variable pairs were prepared; these stimulus orders are shown in Appendix 7. These series were presented first forward, and then backwards. The latter embodied Series Three and Four.

The subject was given a 10 minute rest following the first two series.

Fifteen acceptable observations were collected for each of the two standards.

Treatment of the Data:

Pennies and half-dollars differ in area as a ratio of 1:2. Data for the perceptual effect of divergent size of this ratio are available in the results of Experiment One.

An analysis of variance was conducted in which the results obtained with the monetary stimuli were compared with the results obtained with the plain discs which varied in size as the ratio 1:2. The format of this analysis is shown below:

a n o v a

Source	d.f.
Values	1
Densities	1
V x D	1
Error	56
Total	59

This analysis should detect if the two standard types (Densities) are the source of a significant perceptual effect. Also the analysis should detect if the value factor exerts a significant effect on the perception of numerosity.

As in Experiment One, the distributions of PSEs for both standards were subjected to Chi square analysis where the number of PSEs above the numerosity level of the standard were compared with the number of PSEs below. Because of the lower expected frequencies of these analyses (7.5), Chi square values are reported with and without Yates' correction for continuity.

Results:

Appendix 11 lists PSEs of the first 15 acceptable response protocols for the two monetary standards of Experiment Two. Average PSE for the 30 subjects was 38.75; that is, 37 half-dollars were perceptually equal in numerosity to 38.75 pennies. Four subjects failed to reach the criterion of acceptability on at least one of the four series and were eliminated from the analysis. Of these eliminations three were associated with the B Standard.

Table 4 shows the results of the analysis of variance of the data where the effects of Value and Density were tested for significance. Note that the data observed for the two 2:1 standards of Experiment One are included in this analysis. Accordingly, significance ($p<0.01$) was attributed to the value factor but not to the density factor, nor to the interaction effect of Value with Density.

The distributions of PSEs for both of the value standards of Experiment Two were subjected to Chi square analysis where the number of PSEs above the numerosity level of the standard was compared with the number of PSEs below. Table 5 reports the results of this analysis. The value of Chi square is reported with and

Table 4

Analysis of Variance
Experiment Two: Half-Dollars versus Pennies

Source	df	MS	F
Values	1	9627.00	8.780*
Densities	1	4335.00	3.954
V X D	1	481.00	0.439

*$p<0.01$

Table 5

Chi Square Analysis of Distributions of Observed PSEs, Experiment Two: Half-Dollars versus Pennies

Distribution	Number of PSEs above objectivity	Number of PSEs below objectivity	Value of χ^2 with Yates' correction	Value of χ^2 without Yates' correction
Standard A	11	4	2.40	3.26
Standard B	12	3	4.26*	7.25**
Combined A and B	23	7	7.50**	8.53**

*$p<0.05$

**$p<0.01$

without Yates' correction.

This analysis suggests that the value factor is a highly significant source of numerosity illusion (χ^2=8.53; $p<0.01$); that higher value items appear significantly more numerous than an equivalent quantity of lower value items.

Experiment Three: Dimes versus Pennies

Subjects:

Forty-eight students from the subject pool described for Experiment One served in the experiment.

Stimuli:

The stimuli consisted of concentric patterns of Canadian pennies and dimes mounted with rubber cement on masonite panels (300 mm square) that were coated with dull white latex paint.

Pattern dimensions were chosen such that the 37 dimes (17.9 mm in diameter) of the standard pattern were separated by a diameter of interspace; that is, the ratio of filled to unfilled space within the concentric pattern rings was established as 1:1.

The 10 variables consisted of varying quantities of pennies arranged throughout the same concentric rings as the standard. The range of numerosity and ring composition were the same as for the variables of Experiment One (see Appendix 1).

The variable of equal numerosity as the standard (37), was not included in this experiment. This variable was omitted in order to accommodate better the use of Adaptation Level Theory in the analysis of the illusion. The variable series included the quantities 28, 30, 32, 34, 36, 38, 40, 42, 44 and 46. The AL of this series is taken as 35.05, according to the Helson (1964) equation:

$$\log(AL + 0.75d) = \frac{\sum_{1}^{n} \log x}{n}$$

Procedure:

The subject was read a standard set of instructions (see Appendix 8). The experimental task, as before, involved the identification of the perceptually more numerous stimulus in a standard-variable pair. Pairs were presented simultaneously, unlike the previous experiments, with an exposure duration of approximately two seconds. Exposure duration was not rigorously controlled; using a metronome, the experimenter removed, then replaced a cardboard cover from the pair of stimuli that were placed on a holder approximately half a meter from the subject's eyes.

Two series of the 10 standard-variable pairs were prepared. In these series, spatial and temporal factors were controlled by randomization. Appendix 9 shows the composition of these two series. Subjects were given the second series only if they failed to produce an acceptable protocol on the first series. Subjects who failed to produce an acceptable protocol on the second series were eliminated from the analysis of the data. Thus the criterion of acceptability was slightly higher than in the first two experiments, even though the stimulus series contained one less variable.

Analysis of the Data:

PSEs for each subject were calculated as before. The average PSE for these stimuli was taken as the arithmetic average of all first acceptable response protocols.

The distribution of PSEs observed may be compared with a theoretical distribution of the objective response (POEs) by Sandler's A-test. This statistic denotes significant differences in matched groups; its rigorous derivation from Student's t is given in Sandler (1955) and McGuigan (1960). Its use here, with the comparison of an observed with a theoretical distribution involves the assumption that all subjects are expected to produce an objective response except for the mediation of a numerosity illusion. Therefore, the significance of the illusion itself is taken as the significance of A.

The Chi square comparison of the number of PSEs above objectivity with the number reported below the numerosity level of the standard was carried out.

Results:

Thirteen of the 48 subjects were eliminated from the analysis of the data for failure to reach criterion. Appendix 12 lists the PSEs of the 35 subjects who produced acceptable response protocols. The average PSE was 39.5; that is, the dimes were overestimated. This effect, therefore, cannot be attributed to Adaptation Level (AL=35.05).

Twenty-one subjects reported PSEs above objectivity; seven were below. Chi square of this distribution is significant ($\chi^2=7.00$; $p<0.01$).

Analysis of the distribution of PSEs by Sandler's A test yielded a highly significant value of A=0.0805 ($p<0.001$).

Experiment Four: Random versus Regular

Subjects:

Fifteen students from the subject pool described for Experiment One served in the experiment.

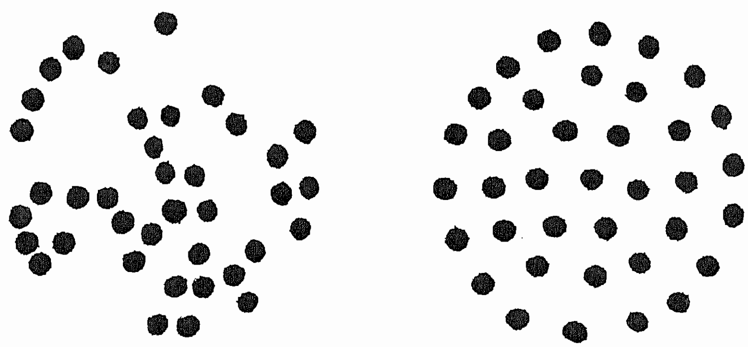
Stimuli:

The 11 variables of Experiment One were used with a "random" standard prepared as follows. Within the outer ring of the variables, four concentric rings were drawn such that the five rings varied in diameter (and therefore in circumference) as the ratios 5:4:3:2:1. These rings were assigned 30, 24, 18, 12, and six positions respectively; these positions on the ring circumferences were determined by the intersection of equiangular radii.

One disc was placed at the pattern centre. The other 36 discs were assigned at random to the 90 possible remaining positions using a table of random numbers. The pattern was photographed from a height of 1950 mm, and thus the discs of this random standard were of equivalent size as the discs of the variables. Figure 4 shows the random standard with the variable

Figure 5

The Random Standard with the Regular Variable
of Equal Numerosity



Random Standard   Concentric Variable

of equal numerosity.

Procedure:

The method of stimulus presentation and the subject's task were the same as in Experiment One. But because of the asymmetrical nature of the standard stimulus, eight different physical positions of the slide were used. No subjects indicated an awareness that the same slide appeared in each pair. This technique was also used by Mokre (1928).

The subject was given the same standard instructions and responded in the same manner as in Experiment One. The same inter-pair and intra-pair exposure durations were used. Fifteen acceptable observations were gathered. Data were analyzed as before.

Results:

The first 15 subjects reached criterion with this standard; no eliminations were recorded. Appendix 13 lists the PSEs of these 15 first acceptable response protocols. Average PSE was 34.3; that is, the random stimulus was underestimated.

Of these 15 observations, 13 PSE were below the numerosity level of the standard, and two were above. Chi square of this distribution, with Yates' correction for continuity, is significant (χ^2=8.066; p<0.01).

Discussion

In the results of these experiments several factors are seen to affect the perception of numerosity in a consistent, systematic manner.

The results of the first experiment must be viewed in the perspective of the previous investigations in order to describe adequately the relationship of item-size to perceived numerosity. Brunswik (1934) reported that large items are perceptually more numerous than small items when the large items are contained in a systematically larger pattern. Mokre (1928) and Courtis (1970) reported that small items are perceptually more numerous than large items when pattern area is equated.

In the present work, more subjects attributed greater numerosity to large items when the large item stimulus was enlarged relative to the overall area of the small item pattern. But when pattern area was equated more subjects attributed greater numerosity to the small items (see Table 3).

To account for these two findings, both pattern size and item-size must be considered, and possibly a third variable, element density, defined as the ratio of filled to unfilled space. Only two degrees of freedom are associated with these three variables; that is, each can be specified in terms of the other two. In the interest of simplicity, the significant effect of differential item-density, hypothesized in the Introduction and confirmed in the Results, can be described best in terms of item-size and pattern area.

It appears that item-size and pattern area both influence perceived numerosity, but in different directions. Numerosity appears to vary inversely with item-size and directly with pattern area. In the present research, this analysis accounts for the results. Under conditions of equal pattern area, the large items were perceptually more numerous; this finding implies the effect of item-size independent of the pattern area factor. But under conditions of unequal area, both factors contribute to the resultant perceived numerosity. Thus the Series B standards were attributed greater relative numerosity than the Series A patterns.

Presumably item-size contributed a negative influence on the resultant numerosity of the Series B patterns since it was found to do so under conditions of equal pattern area. But because the enlarged standards were attributed greater numerosity by the majority of subjects it is necessary to postulate that numerosity varies directly with pattern area. Further, to account for these results, it is necessary to postulate that the contributions of item-size and pattern area must be algebraically summated in the prediction of the perceptual numerosity of a given quantity of objects.

It is important to note that the results of the present experiment did not replicate to the same degree, Brunswik's effect--the overestimation of large items under conditions of enlarged pattern area for the large item stimulus. The average PSE for the Series B standards of the present research was 37.0; that is, right on objectivity. The average PSE in the Brunswik research was substantially different from objectivity (34 large items were perceptually equal to 40 small items). This finding is accounted for as follows. The large item patterns of the present experiment were not enlarged relative to the small item patterns to the same extent as Brunswik's large item stimulus. Therefore the pattern area effect observed in this experiment was not as powerful as the effect Brunswik observed.

The randomness variable, investigated in Experiment Four offers much potential for further quantitative investigation. The significant tendency for the random distribution of elements to appear less numerous than a concentric ring arrangement of an objectively equivalent quantity of elements may be explained in two ways.

First, it could be that the significant effect observed with the random standard is in fact a consequence of the heterogeneous distribution of elements. But secondly, it is also possible that the effect is a result of the pattern area factor described above. The pattern formed by the random standard has a smaller total area than the regular variables--approximately one fourth smaller by area. Therefore the underestimation of the random standard may have been due to its smaller relative pattern area. It seems reasonable to predict that a random pattern with larger pattern area than the regular concentric patterns would be overestimated in numerosity. This finding was, in fact, reported thirty years ago by Taves (1941).

Taves (1941) conducted a similar investigation to determine the effect of configuration on perceived numerosity. Using a variation of the method of constant stimuli, the method of half estimation, Taves observed that when the discs in a stimulus field were arranged into compact circles a significant decrease in perceived numerousness relative to a field of randomly scattered discs resulted. Taves' stimuli are reproduced in Figure 6.

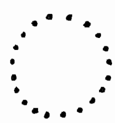
Taves interpreted this decrease in numerosity as being a result of the circular configuration. But it is immediately apparent in the light of the present discussion that Taves' circular stimulus is clearly smaller than the patterns with which it was compared. Therefore his finding that the more compact stimulus was perceptually less numerous is not surprising and can be accounted for by the hypothesis of this paper; that is, that numerosity is directly proportional to pattern size.

It appears then, that heterogeneity of pattern density was not an isolated variable in the Taves (1941) experiment nor in the present research. Further work will be required before the effect of that factor on perceived numerosity can be determined. It is essential that the pattern size variable be brought under control in the comparison of random and even-density patterns before the true nature of the effect of randomness can be determined.

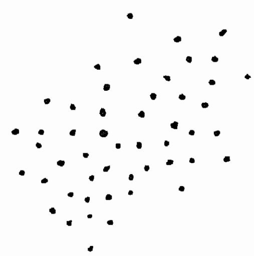
Several systematic degrees of randomness could be generated by the present method by varying the number of concentric rings which support the potential positions of stimulus items. In the random pattern of the present experiment, five rings were used:

Figure 6

The Stimuli of the Taves (1941) Experiment on the Effect of Configuration on Perceived Numerosity



Regular Stimulus
(20 dot circle)



Random Stimulus
(50 dot standard)

the more rings, the more positions available for random assignment, and the more potential positions, the greater the homogeneity of distribution.

It should therefore be possible to generate several systematic degrees of randomness by this method, and ultimately to develop the function of the effect of increasing heterogeneity of item distribution on perceived numerosity.

The Brunswik/Fazil demonstration of the value--numerosity illusion is corroborated by the results of Experiments Two and Three, and it appears reasonable to conclude that higher value items are perceptually more numerous than equivalent quantities of lower value items.

But the Brunswik (1956) work with the value illusion was not complete. In the Brunswik/Fazil experiment the higher value items were arranged in a pattern that was systematically larger than the pattern composed of the lower value items. Since the coins were of different sizes, it was necessary for him to do this so as to preserve elemental density equivalence between patterns.

The present research indicates that enlarging pattern area accounts for a resultant increase in perceived numerosity. Furthermore, Brunswik was aware that when analogous patterns of valueless discs were compared, the large items which in his method composed the larger pattern were perceptually more numerous. In the Zuk-Kardos experiment 34 large items were perceptually equal to 40 smaller discs. But the ratios of size divergence of the discs of the Brunswik/Zuk-Kardos research and coins of the Brunswik/Fazil work were not the same. Therefore the strongest claim that Brunswik (1956) made concerning the effect of value on numerosity was that the overestimation of larger items was greater when the larger items were also more valuable. The Brunswik research did not demonstrate a value-numerosity illusion where the item-size/pattern area effects were completely under control.

In the present research, the item-size/pattern area effects were controlled in two ways. First, in Experiment Two (half-dollars versus pennies) the value factor was isolated statistically in the analysis of variance. In this experiment, valueless discs which differed in size in the same ratio as the coins were compared for numerosity, as well as the coins themselves, and the value factor was analyzed as a main effect.

Secondly, in Experiment Three (dimes versus pennies) the value effect was isolated by the virtual elimination of item-size/ pattern area considerations since these coins are approximately equal in size. And since dimes are actually slightly smaller, it is concluded that the value factor alone accounts for a numerosity illusion in its own right and not just as an interaction effect with size.

An interesting variation of these experiments would involve British subjects comparing larger pennies with smaller sixpence coins. But since Britain has recently adopted decimal currency and has introduced new coinage of conventional sizes, that comparison is no longer possible. Presumably, size and value in this comparison would counteract each other, and whether or not the value illusion would be apparent under these conditions would have been an interesting empirical question.

Ansbacher (1937; 1938) reported data which gives evidence of the value illusion which is especially relevant to the present research. In the Ansbacher study, stamps of equal size but different value were compared for numerosity and a significant tendency for the higher value items to be overestimated was found. Ansbacher (1937) used Canadian stamps with Canadian subjects, and American stamps with American subjects. In each case, 20 subjects compared variable patterns of 3¢ stamps with a standard of thirty 2¢ stamps.

Ansbacher found that the value--numerosity illusion did not appear spontaneously in his experiment. In order to get his subjects to perceive the more valuable stamps as more numerous than the less valuable stamps, it was necessary for him to induce the illusion by having the subjects first compare quantities of 2¢ and 3¢ stamps for value.

Ansbacher's statistical analysis did not test the significance of the illusion itself. He tested for the significance of differences between mean PSEs found with American and Canadian subjects, with American and Canadian stamps, and for different degrees of familiarity with the stamps.

But he did provide the modern reader with enough information about his observations to test his results for significance. Table 6 lists the mean PSEs and standard deviations reported by Ansbacher for the 20 American and 20 Canadian subjects, both before and after the value inducing task. It seems reasonable to combine the American subjects with the Canadians since each group performed similar numerosity comparisons with similar, familiar stimuli. This operation yields a mean PSE of 30.31 and standard deviation of 1.20 for the 40 subjects before the value inducing task (that is, 30.31 three cent stamps were perceptually equal to 30 two cent stamps).

This distribution may be compared with a hypothetical, correlated

Table 6

Results of the Ansbacher (1937) Experiment

	20 Americans		20 Canadians		40 Subjects Combined	
	Mean PSE	SD	Mean PSE	SD	Mean PSE	SD
Before Value Induction	30.52	1.00	30.09	1.34	30.31	1.20
After Value Induction	29.19	1.24	28.71	1.64	28.95	1.47

distribution of the objective response (mean PSE=30; SD=0). The legitimacy of this comparison is suggested by McNemar (1959; p 107). This test yields a nonsignificant value of Student's t (t=1.63) for 39 degrees of freedom. It therefore appears that Ansbacher's subjects did not experience a statistically significant value-numerosity illusion before the value inducing task.

Combining the observations for the Americans and Canadians after the value inducing task yields a mean PSE of 28.95 with standard deviation of 1.47; that is, 28.95 three cent stamps were perceptually equal to 30 two cent stamps. When this distribution is compared with a correlated distribution of the objective response (mean PSE=30; SD=0) a significant value of Student's t is found (t=4.46; p<0.001).

It is therefore reasonable to conclude that Ansbacher did observe a statistically significant value-numerosity illusion, but only after the value inducing task. Before such a task, Ansbacher's subjects showed no significant illusion.

Ansbacher interpreted his pre-induction set results in terms of <u>overcompensation</u>. That is, Ansbacher theorized that subjects overestimate the lower value items as a result of over-reaction to their awareness of the influence of value. But after the inducing task, the effect of value on perceived numerosity became apparent in the displaced PSE. Ansbacher attributed this theory to Brunswik (1934).

Ansbacher provided evidence that the Brunswik/Fazil value illusion was also an induced effect. Ansbacher (1937) cited personal communication with Brunswik to suggest that subjects in the Fazil experiment were given the task of comparing coin patterns for value equivalence as well as for numerosity in the same experimental session. It therefore appears that the value illusion reported by Brunswik (1956) was at least partially the result of induction by the side comparison for value equivalence.

In the present research, the significant value illusion was found without induction. Subjects were required to compare the coin patterns for numerosity only, and the illusion was observed despite the fact that the subjects were not primed to the influence of the value factor. It is believed that these results represent the first empirical demonstration of the uninduced value illusion.

It is, however, recognized that the subjects of the present research may have been set to experience the value illusion despite the lack of formal induction in the experimental procedure. The instructions identified the coins according to monetary value, and the subjects responded by writing "pennies " or "dimes" ("p" or "q" in Experiment Two) on the score sheets. Whether or not the illusion would be apparent under the conditions where the coins were not identified as to value and the subjects responded by merely pointing to the pattern which appeared more numerous is an interesting empirical question, the answer to which will throw new light on the role of induction in the value illusion.

Nelson and Lechelt (1970) also reported a value illusion of some relevance to the present research. In their study, subjects estimated the apparent numerosity of patterns of coins and of slugs on an absolute basis. It was found that the valuable items were estimated to be significantly greater in numerosity than an equivalent quantity of slugs. This finding is compatible with the results of the present experiment; by the mediation of some cognitive process, the numerosity of valuable items is overestimated. Nelson and Lechelt (1970) also demonstrated quantitative differences in the overestimation in children from different socio-economic backgrounds.

A theory of cognitive factors accounting for the value--

numerosity illusions appears to be compatible with the data of all the previous value experiments described here as well as with the results of the present research.

Taken together, the four experiments of this thesis are interesting inasmuch as several significant effects in the perception of numerosity are empirically demonstrated. Very little theoretical discussion of the numerosity illusions is available in the literature. Nelson and Bartley (1961) gave some attention to numerosity in a general sense but their paper dealt more with absolute numerosity. Some discussion of the illusions of relative numerosity is offered in Ansbacher (1937).

Ansbacher cited Kasting (1935) as the originator of a theory of over-compensation which seemed to account for the different item-size illusions in children and in adults. Kasting replicated the Binet illusion in children, but found the item-size effect in adults to be of the qualitative nature described by Mokre (1928); that is, children overestimate large items while adults underestimate large items.

Kasting (1935) proposed the following explanation, as paraphrased in Ansbacher (1937):

...the stimuli when presented are seen at first

> as a whole, of which the number of elements is just one part-characteristic. The task in number comparison is to abstract this one characteristic from others like size, density of the distribution, etc. The knowledge that size of the elements and density must be taken into account, together with the intention to do so, may lead to over-compensation for the expected error and therefore to a numerical underestimation of the group which contains the larger elements. This knowledge and with it over-compensation are the stronger, the larger the difference between the two groups which are to be compared (Ansbacher, 1937, p 31).

Thus, Kasting's theory of the item-size illusion was analogous to the Brunswik/Ansbacher theory of the value effect. Both theories presume some form of cognitive interaction of the subject's "knowledge" with the sensory information. Children apparently respond directly to the sensory information, since they lack the sophistication of adults.

But the research on the item-size/pattern area interaction is not complete and more work will be necessary before a completely sound theoretical interpretation of the size illusion is possible. Future experimentation should examine the effects of pattern size with equal size items. By using extremely small items it should be possible to examine the pattern size factor in isolation from the effects of item-size differential and from the effects of differential element density (the ratio of filled to unfilled space within the overall pattern surface).

The use of very small items will not physically eliminate element density differential between different size patterns of equal size items, since even very small density ratios are decreased proportionately as pattern size increases. But the psychological effect of density differential between patterns of tiny stimuli--pin point light sources--distributed throughout relatively large overall areas may be perceptually inconsequential.

Mokre (1928) reported the results of an experiment on the pattern size--numerosity relationship, with equal size items, and concluded that numerosity varied inversely with pattern size. That is, Mokre concluded that the pattern size--numerosity illusion was qualitatively opposite to the conclusion of this thesis: that numerosity varies directly with pattern size.

But Mokre used relatively large stimulus items and his results must therefore reflect the extraneous effect of density differential as well as pattern size. Furthermore his work involved only seven subjects and he did not analyze his data statistically.

When Mokre's data, shown in Appendix 14, are subjected to analysis by Sandler's A test, a non significant value for A is found for the pattern area effect. By the same analysis, his data for the item-size effect are also found to be non significant.

It seems reasonable to conclude that Mokre's results do not represent a substantial refutation of the conclusion of this thesis regarding the pattern area--numerosity effect; but the necessity of further study is indicated.

It is suspected that the apparent size of stimulus items is intimately related with item density. When equivalent quantities of large and small stimuli are distributed evenly over equal areas, the larger stimuli are naturally more densely packed together. In Brunswik's method, element density was maintained constant by having the overall pattern diameter vary as item diameter. In Brunswik's work the larger pattern was perceived as more numerous. Since Brunswik's standard and variables were effectively similar stimuli (the standard was analogous to a photographic blow-up of the variable) the one may be interpreted perceptually as being merely closer than the smaller pattern. The Brunswik illusion may therfore be associated with size-distance constancy phenomena.

In the stimuli of the present research, the standard and variables are clearly different; that is, the standard is clearly not just a variable observed at a closer distance, because of obvious differences in item-density between patterns. It is suspected that this stimulus property precludes the possibility of the subject interpreting a size-distance situation, since respective overall pattern size does not vary as item-size. Brunswik presumed, apparently, that item-density equivalence between standard and variables would yield the isolation of the item-size--numerosity effect. It is the contention of this thesis that it does not. In Brunswik's experiment, the item-size--numerosity effect is confounded with size-distance.

But of course, in the stimuli used here, the item-size--numerosity effect is confounded with item-density. Further experimentation with the problem is required before much certainty may be ascribed to any interpretation of the illusion.

Further research may find still other factors influencing the perception of numerosity. The very fact that subjects can accurately perform value--numerosity, and size--numerosity comparisons and produce reasonable consistent responses gives substance to the contention that numerosity is a primary stimulus property, just as size, shape, colour and brightness. Further study should examine the possible interactions of numerosity with these other primary properties of stimuli.

Shape--numerosity effects could be investigated, for example, by comparing triangular patterns with circular patterns or with square patterns.

Colour--numerosity effects could be examined by having subjects compare the numerosity of a pattern composed of items of one colour with a pattern composed of items of another colour. Since colour and brightness are known to interact as in the visibility curve, so it is possible that colour and numerosity are also interacting variables. Red items may appear as significantly different in numerosity from an equivalent quantity of blue items.

But work with the colour-numerosity effect would have to control for brightness which has already been implicated as an important factor in the perception of numerosity. Hunter and Sigler (1940) demonstrated that the span of visual discrimination (the total number of items that subjects can accurately discriminate in tachistoscopic presentation) varies in accordance with the Bunsen-Roscoe Law. Lechelt and Nelson (1971) have pointed out that the estimation of numerosity in stimulus arrays of physical quantity well in excess of the span of visual discrimination, also follows the Bunsen-Roscoe Law.

Lechelt and Nelson (1971) used physical quantities ranging from 2 to 128 items and found that the perceived numerosity reported by their subjects was also in accordance with what would be predicted by the Ixt = C equation; the experience of numerosity varies directly with the total energy of stimulation.

In conclusion, the prospects of uncovering other illusion effects in the perception of numerosity seem promising.

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Appendices

Appendix 1

Composition by Ring of the 11 Variables
Experiment One

Variable	Outer Ring	Intermediate Ring
28	11	10
30	13	10
32	14	11
34	16	11
36	17	12
37	18	12
38	19	12
40	20	13
42	22	13
44	23	14
46	25	14

Appendix 2

Element Densities and Relative Size of the Standard and Variables, Experiment One

Stimulus	Element Density (ratio of filled to unfilled space)	Actual pattern diameter of outer ring	Camera[1] height	Projected size of outer ring (diameter)	Visual[2] angle outer ring	Projected size of items (diameter)	Visual angle items
2:1 (A)	20:30	273.01	1380 mm	440 mm	11°10'	30 mm	0°46'
2:1 (B)	34:62	305.58	1380 mm	500	12°38'	30	0°46'
4:1 (A)	40:30	191.11	975	450	11°30'	45	1°10'
4:1 (B)	34:44	248.28	975	600	15°10'	45	1°10'
9:1 (A)	60:10	127.40	650	480	12°10'	72	1°50'
9:1 (B)	34:30	203.58	650	780	19°38'	72	1°50'
Variable	20:50	382.21	1950	430	10°56'	20	0°32'

[1] Camera heights were erroneously measured from the pattern surface to the focal plane rather than to the lens centre. Consequently, actual sizes of items and of patterns do not conform precisely with theoretical sizes. The items of the standards are slightly larger. The three Series A standards should theoretically have a projected outer ring diameter equal to the projected outer ring diameter of the variable. However, in the actual patterns, there is a systematic tendency for the standards of greater item-size differential to be increasingly larger than the variables.

[2] Visual angles were calculated for an eye-stimulus distance of 2.25 meters. The projector was situated 2 meters from the screen and the subject sat beside the projector with his eyes approximately 250 mm behind the lens.

Appendix 3

Standard Instructions: Experiment One

This is an experiment in the perception of number. In a moment you will see a series of 15 pairs of slides. Each slide depicts white dots on a dark background. You are to decide which slide in each pair contains the greater number of dots. If you feel the first slide of a given pair contains the greater number of dots you will respond by writing "one" on the score sheet. If you feel the second slide of a given pair contains the greater number of dots you will respond by writing "two".

Do not try to count the dots in any of the patterns; just try to get an overall impression of number.

You must respond to each pair with a choice; that is, you cannot respond by calling any pair the same.

Any questions?

(The following abbreviated instructions were read to the subject just prior to Series Two and again just prior to Series Three)

This time you will see another series of 11 pairs of slides. Remember, you are to respond to each pair by deciding which slide contains the greater number of dots.

Do not try to count the dots in any of the patterns. Just try to get an overall impression of number.

Appendix 4

Stimulus Orders Within the Four Series of the Eleven Standard-Variable Pairs, Experiment One

Series One[1]		Series Two		Series Three		Series Four	
1st Slide	2nd Slide	1st Slide	2nd Slide	1st Slide	2nd Slide	1st Slide	2nd Slide
46	28						
S	44						
30	S						
32	42						
S	30	44	S	40	S	S	36
42	S	28	S	38	S	38	S
S	37	S	32	28	S	S	46
34	S	46	S	S	32	S	28
S	38	S	38	37	S	32	S
S	40	S	34	S	46	S	44
S	28	30	S	S	36	34	S
36	S	S	37	S	34	S	42
46	S	36	S	44	S	37	S
S	44	42	S	30	S	S	30
32	S	S	40	S	42	40	S

[1]The first four pairs of Series One constituted the warm-ups; responses to these pairs were not considered in the computation of the subject's PSE

Appendix 5

Composition by Ring of the 15 Variables
Experiment Two

Variable	Outer Ring	Intermediate Ring
28	11	10
30	13	10
32	14	11
34	16	11
36	17	12
37	18	12
38	19	12
40	20	13
42	22	13
44	23	14
46	25	14
48	26	15
50	28	15
52	29	16
54	31	16

Appendix 6

Standard Instructions: Experiment Two

This is an experiment in the perception of number. In a moment you will be shown a series of 15 pairs of slides. The slides depict patterns of coins--quarters and pennies. That is, a pair will consist of a pattern composed of pennies, and a pattern composed of quarters. The pairs will be presented successively--that is, you will see first one slide of a given pair, and then the other.

If you feel there are more pennies in a given pair, you will respond by writing "p" on the score sheet. If you feel there are more quarters in a given pair, you will respond by writing "q". That's "p" for more pennies; "q" for more quarters.

Do not try to count the coins in any of the patterns; just try to get an overall impression of number.

You must respond to each pair with a choice--that is, you cannot respond by calling any pair the same.

Any questions?

(The following abbreviated instructions were read to the subject just prior to Series Two and again just prior to Series Three)

Now I shall show you another 15 pairs. Remember, the experiment deals with the perception of number. If you feel there are more pennies, mark "p"; if you feel there are more quarters, mark "q".

Do not count the coins--just try to get an overall impression of number.

Appendix 7

Stimulus Orders Within the Four Series of the 15 Standard-Variable Pairs, Experiment Two*

Series One		Series Two		Series Three		Series Four	
1st Slide	2nd Slide	1st Slide	2nd Slide	1st Slide	2nd Slide	1st Slide	2nd Slide
42	S	S	38	38	S	S	34
30	S	30	S	50	S	32	S
37	S	S	42	48	S	46	S
S	54	48	S	S	28	S	37
S	40	40	S	S	34	28	S
52	S	S	52	46	S	S	50
32	S	S	44	S	44	S	36
S	36	S	54	36	S	54	S
44	S	36	S	S	32	44	S
S	46	50	S	S	52	52	S
34	S	S	28	40	S	S	40
28	S	37	S	54	S	S	48
S	48	S	46	S	37	42	S
S	50	S	32	S	30	S	30
S	38	34	S	S	42	38	S

*Note: Series Three and Four are the reverse of Series One and Two respectively. Only two sets of slides were prepared. These were run through the projector forward for Series One and Two, and backwards for Series Three and Four.

Appendix 8

Standard Instructions: Experiment Three

This is an experiment in the perception of number; in a moment you will be shown two sets of coins: a set of pennies and a set of dimes.

Sometimes there will be more pennies, and sometimes there will be more dimes. You are to decide whether you see more pennies ar more dimes.

If you feel there are more pennies, write "pennies" on the answer sheet. If you feel there are more dimes, write "dimes".

Do not try to count the coins in any of the patterns; just try to get an overall impression of number.

You must respond to each trial with a choice; that is, you cannot respond by calling any trial the same. If you are not sure, guess.

(The following abbreviated instructions were read to the subject just prior to Series Two and again just prior to Series Three)

Now you will be shown another 10 pairs. Remember, the experiment deals with the perception of number. If you feel there are more pennies, write "pennies"; if you feel there are more dimes, write "dimes".

Appendix 9

Stimulus Orders Within the Two Series of the 10 Standard-Variable Pairs, Experiment Three*

Series One		Series Two	
Left	Right	Left	Right
S	30	44	S
42	S	28	S
34	S	S	32
S	38	46	S
S	40	S	38
S	28	S	34
36	S	30	S
46	S	S	36
S	44	42	S
32	S	S	40

*Note: Standard composed of 37 dimes

Appendix 10

Observed PSEs of the 15 First Acceptable Response Protocols for Each of the Six Standards of Experiment One

	Standard					
	2:1 (A)	2:1 (B)	4:1 (A)	4:1 (B)	9:1 (A)	9:1 (B)
	31.0	33.0	33.0	31.0	33.0	31.0
	33.0	33.0	33.0	33.0	33.0	31.0
	33.0	33.0	33.0	33.0	35.0	33.0
	33.0	33.0	33.0	33.0	35.0	33.0
	33.0	36.5	35.0	35.0	35.0	37.5
	35.0	37.5	36.5	36.5	36.5	37.5
	35.0	37.5	36.5	36.5	36.5	37.5
	35.0	37.5	36.5	36.5	36.5	37.5
	35.0	39.0	37.5	36.5	36.5	37.5
	35.0	39.0	39.0	37.5	37.5	39.0
	35.0	39.0	39.0	37.5	37.5	39.0
	35.0	39.0	39.0	39.0	37.5	39.0
	37.5	39.0	41.0	39.0	37.5	41.0
	37.5	41.0	41.0	41.0	39.0	41.0
	43.0	43.0	43.0	43.0	41.0	43.0
Mean PSE	35.1	37.3	37.0	36.5	36.5	37.1

*Note: Twelve subjects failed to reach the criterion of Acceptability, and were eliminated from the analysis of the data. The number of eliminations for each standard was as follows:

2:1 (A)	5
2:1 (B)	0
4:1 (A)	4
4:1 (B)	0
9:1 (A)	0
9:1 (B)	3

Appendix 11

Observed PSEs of the 15 First Acceptable Response Protocols for the Two Monetary Standards of Experiment Two

	Standard A	Standard B
	31.0	35.0
	31.0	35.0
	33.0	36.5
	36.5	37.5
	37.5	37.5
	37.5	39.0
	37.5	39.0
	37.5	39.0
	39.0	41.0
	39.0	41.0
	39.0	41.0
	41.0	41.0
	41.0	41.0
	45.0	43.0
	47.0	43.0
Mean PSE	38.2	39.3

Appendix 12

Observed PSEs of the 35 First Acceptable Response Protocols, Experiment Three

	45
	45
	45
	43
	43
	43
	43
	43
	43
	43
	43
	43
	41
	41
	41
	41
	41
	39
	39
	39
	39
	37
	37
	37
	37
	37
	37
	37
	35
	35
	35
	35
	35
	35
	33
Mean PSE	39.5

Appendix 13

Observed PSEs of the 15 First Acceptable Response Protocols, Experiment Four

	31.0
	31.0
	33.0
	33.0
	33.0
	33.0
	33.0
	33.0
	35.0
	35.0
	36.5
	36.5
	36.5
	37.5
	37.5
Mean PSE	34.3

Appendix 14

Results of the Mokre (1928) Experiments

Pattern Size Experiment

Variable smaller than Standard	Variable larger than Standard	D	D^2
13.93	14.83	+0.90	0.810
14.95	15.10	+0.15	0.025
15.50	14.28	-1.22	1.488
15.93	14.13	-1.80	3.240
13.95	15.90	+1.95	3.803
14.65	14.90	+0.25	0.063
14.85	15.30	+0.45	0.203
		$\Sigma D=+0.63$	$\Sigma D^2=9.632$

Sandler's $A=\frac{\Sigma D^2}{(\Sigma D)^2}=$ 28.6 (not significant)

Item Size Experiment

Variable items smaller than Standard items	Variable items larger than Standard items	D	D^2
14.08	14.18	+0.10	0.010
14.18	15.88	+1.70	2.890
14.68	14.80	+0.12	0.014
14.65	14.75	+0.10	0.010
14.45	15.65	+1.20	1.440
14.00	15.90	+1.90	3.610
15.00	14.75	-0.25	0.063
		$\Sigma D=+4.87$	$\Sigma D^2=8.037$

Sandler's $A=\frac{\Sigma D^2}{(\Sigma D)^2}=0.338$ (not significant)